Shape Moments for Region-Based Active Contours

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Goal

- Introduce shape prior into the Chan and Vese model

Improve performance in the presence of:

- Occlusion
- Cluttered background
- Noise

Overview

- Region-based active contours
  - The Mumford-Shah model
  - The Chan and Vese model
  - Level-set function
- Shape moments
  - Geometric moments
  - Legendre moments
  - Chebyshev moments
- Segmentation with shape prior
- Experimental results

Mumford-Shah model

- D. Mumford, J. Shah in 1989
- General segmentation model
  - o • •R^2, u_0-given image, u-segmented image, C-contour

$$E_{MS}(u,C) = l^2 \int_{\Omega} (u_0 - u)^2 \, dx + \int_{\Omega \setminus C} \|\nabla u\|^2 \, dx + m\,|C|$$

1   2   3

1. Region similarity
2. Smoothness
3. Minimizes the contour length

Chan and Vese model I.

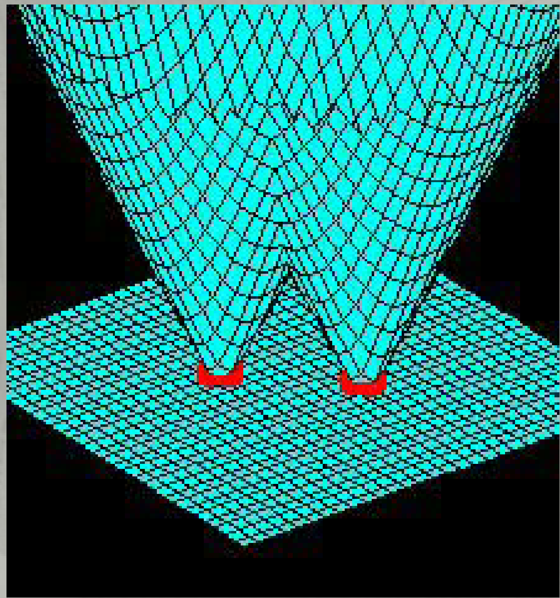
- Intensity based segmentation
- Piecewise constant Mumford-Shah energy functional (cartoon model)
  - Inside (c_1) and outside (c_2) regions

$$E_{CV}(c_1, c_2, C) = \lambda_1 \int_{\Omega_{in}} (u_0 - c_1)^2 dx + \lambda_2 \int_{\Omega_{out}} (u_0 - c_2)^2 dx + |C|$$

- Active contours without edges [Chan and Vese, 1999]
  - Level set formulation of the above model
  - Energy minimization by gradient descent

Level-set method

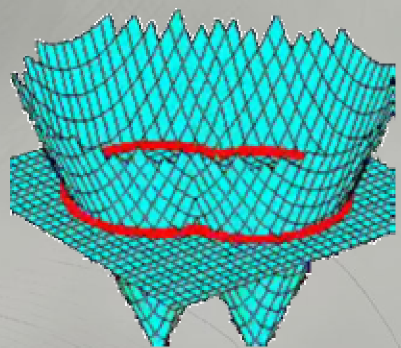
- S. Osher and J. Sethian in 1988
- Embed the contour into a higher dimensional space
  - Automatically handles the topological changes
  - f(., t) level set function
  - Implicit contour (f = 0)
  - Contour is evolved implicitly by moving the surface f

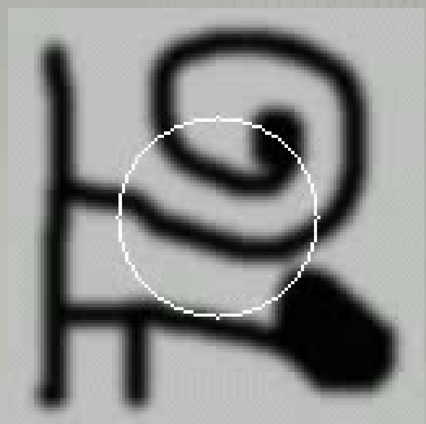


Chan and Vese model II.

- Level set segmentation model
  - Inside $\phi>0$; outside $\phi<0$
  - H(.)-Heaviside step function
  - It is proved in [Chan & Vese, '99] that a minimizer of the problem exist

$$E_{CV}(c_1,c_2,\phi)=\lambda_1\int_\Omega(u_0-c_1)^2H(\phi)dx+\lambda_2\int_\Omega(u_0-c_2)^2(1-H(\phi))dx+\nu\int_\Omega|\nabla H(\phi)|\,dx$$





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Geometric shape moments

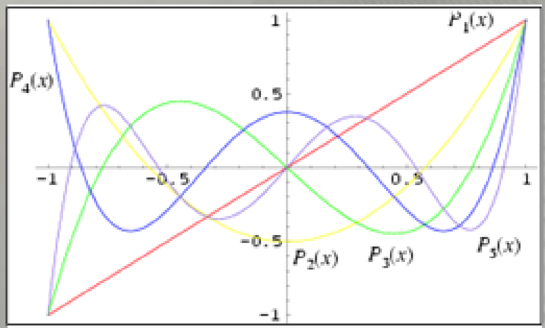
- Introduced by M. K. Hu in 1962
- Normalized central moments (NCM)
  - Translation and scale invariant
  - (x_c, y_c) is the centre of mass (translation invariance)

$$\eta_{pq} = \iint \frac{(x - x_c)^p (y - y_c)^q}{M_{00}^{(p+q+2)/2}} dxdy$$

Area of the object (scale invariance)

Legendre moments

$$l_{pq} = \frac{(2p+1)(2q+1)}{4} \int_{-1}^{1}\int_{-1}^{1} P_p(x) P_q(y) f(x,y)\,dx\,dy$$



- Where $P_p(x)$ are the Legendre polynomials
- Orthogonal basis functions

o Provides a more detailed representation than normalized central moments:

Shape NCM (h) Legendre (l)

NCM is dominated by few moments while Legendere values are evenly distributed

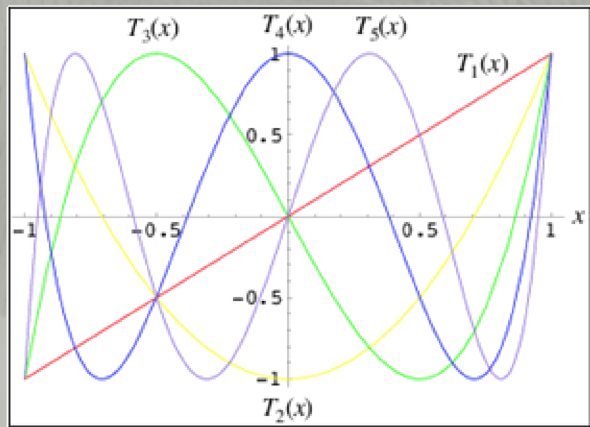
Chebyshev moments

- Ideal choice because discrete

$$T_{mn} = \frac{1}{\mathrm{r}(m,N)\,\mathrm{r}(n,M)} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} T_m(i) T_n(j) f(i,j)$$

  - Where, *f* (n, N) is the normalizing term, $T_m(.)$ is the Chebyshev polynomial
- Can be expressed in term of geometric moments

Chebyshev polynomials:



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New energy function

- We define our energy functional:

$$E(c_1, c_2, \phi, \mu_{ref}) = E_{CV}(c_1, c_2, \phi) + E_{prior}(\mu_{ref}, \phi)$$

- Where E_{prior} defined as the distance between the shape and the reference moments

$$E_{prior}(\mu_{ref}, \phi) = \sum_{p,q}^{p,q \leq N} (\mu^{pq} - \mu^{pq}_{ref})^2$$

  - μ^{pq} shape moments

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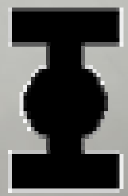
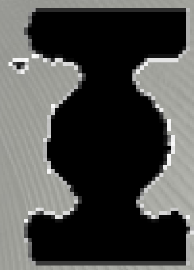
Geometric results

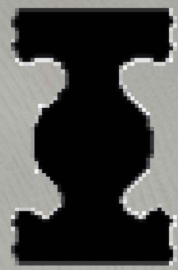


Reference object

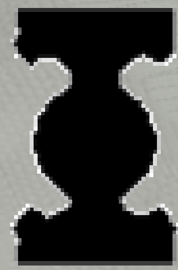
Legendre moments



p, q • 12

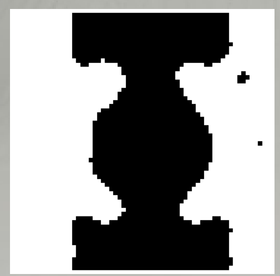


p, q • 16

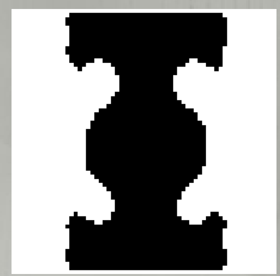


p, q • 20

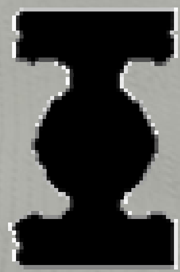
Chebyshev moments



p, q • 12



p, q • 16



p, q • 20

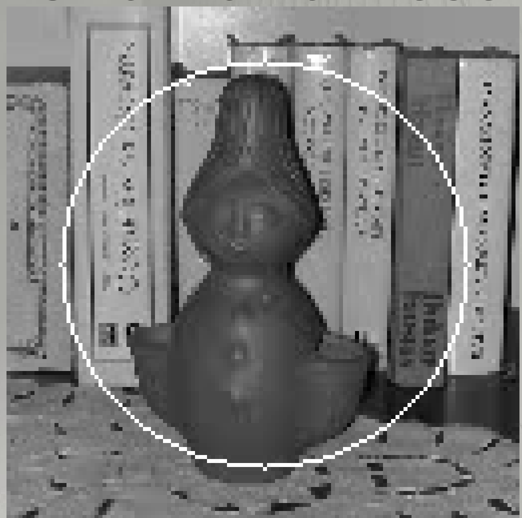
Result on real image

Original image

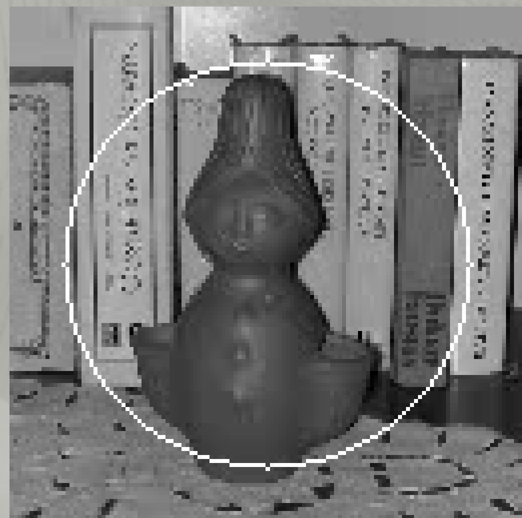
+

Reference object

Chan and Vese



Chan and Vese with shape prior



Conclusions, future work

- Legendre is faster
- Chebyshev is slower but it's discrete nature gives better representation

- Future work:
  - Extend our model to Zernike moments
  - Develop segmentation methods using shape moments and Markov Random Fields

Thank you!

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