Skeletonization and its applications

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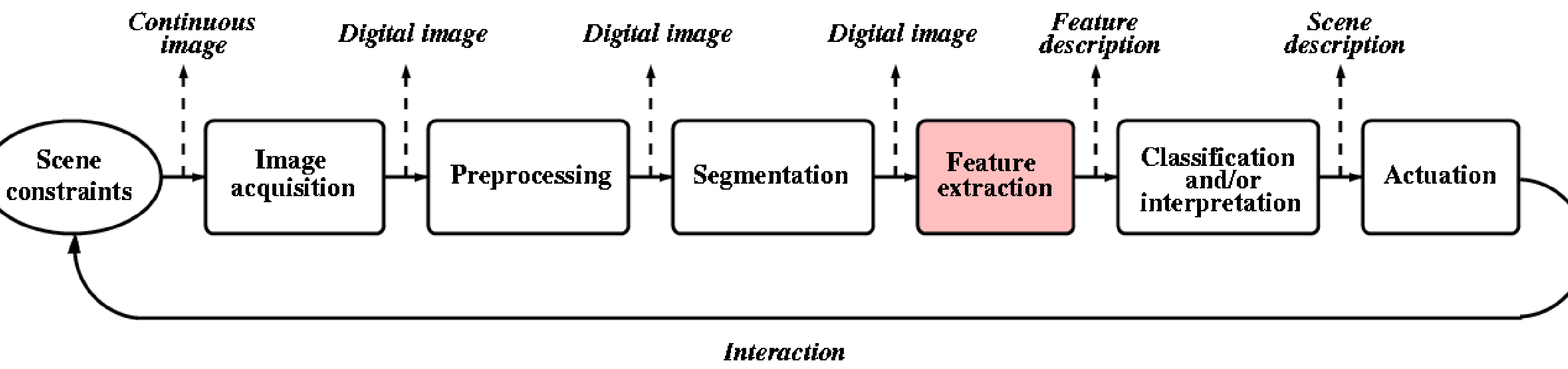
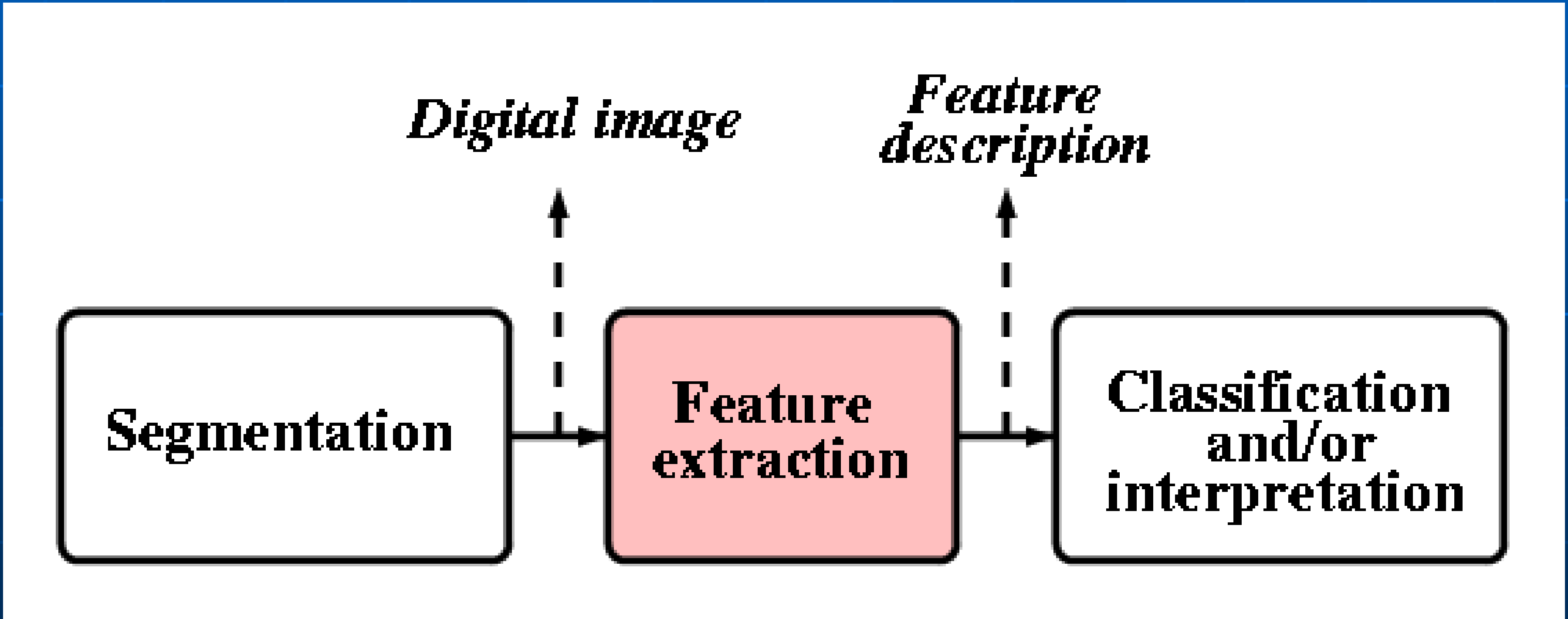
The generic model of a modular machine vision system



(G.W. Awcock, R. Thomas, 1996

Feature extraction – shape representation



(G.W. Awcock, R. Thomas, 1996

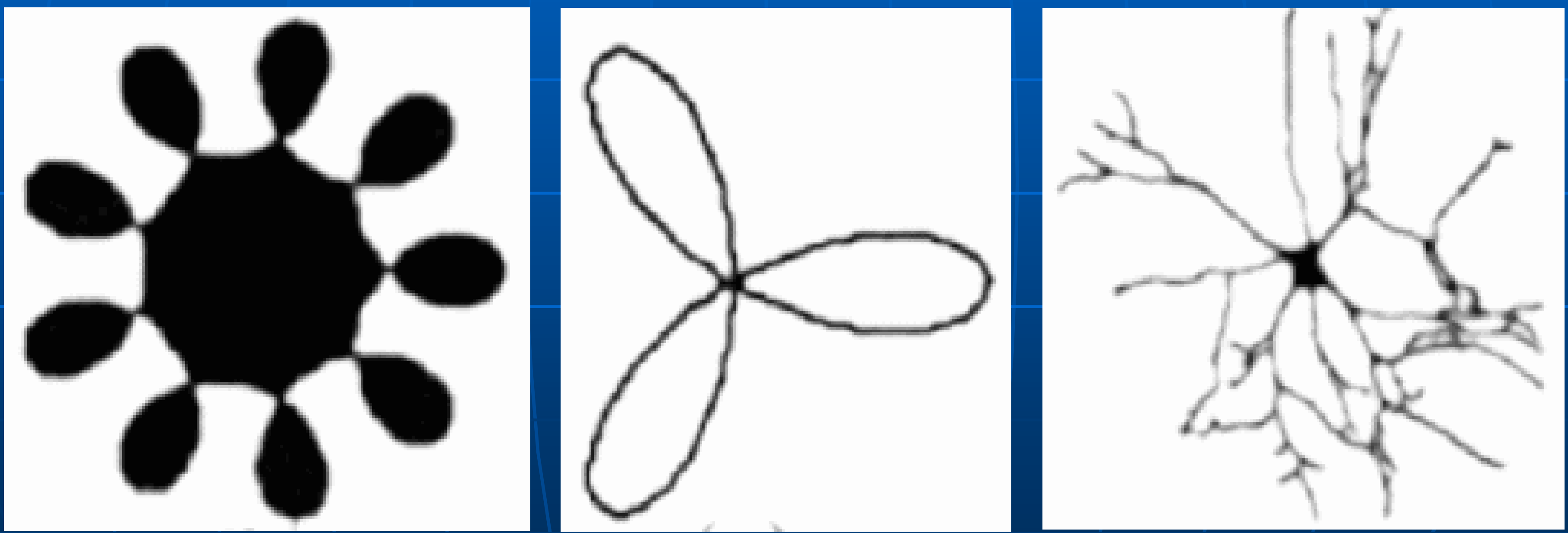
Shape

It is a fundamental concept in computer vision.

It can be regarded as the basis for high-level image processing stages concentrating on scene analysis and interpretation.

Shape

It is formed by any connected set of points.



Examples of planar shapes

(L. F. Costa, R. Marcondes, 2001

Shape representation

- to describe the <u>boundary</u> that surrounds an object,
- to describe the <u>region</u> that is occupied by an object,
- to apply a <u>transform</u> in order to represent an object in terms of the transform coefficients.

Contour-based shape representation

- chain-code
- run-length
- polygonal approximation
- syntactic primitives
- spline
- snake / active contour
- multiscale primitives

(L. F. Costa, R. Marcondes, 2001

Region-based shape representation

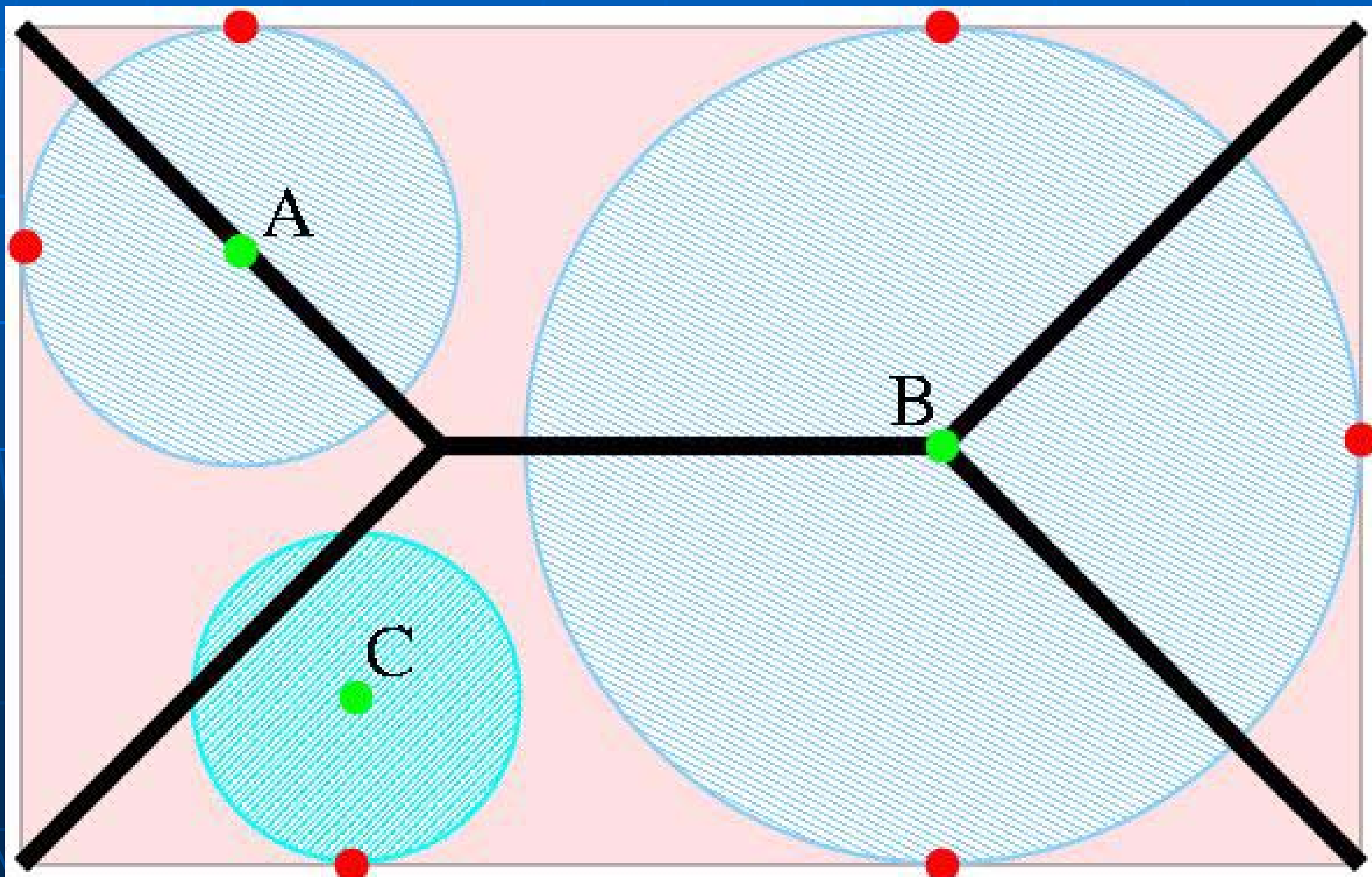
- polygon
- Voronoi / Delaunay
- quadtree
- morphological decomposition
- convex hull / deficiency
- run-length
- distance transform
- **skeleton**

(L. F. Costa, R. Marcondes, 2001

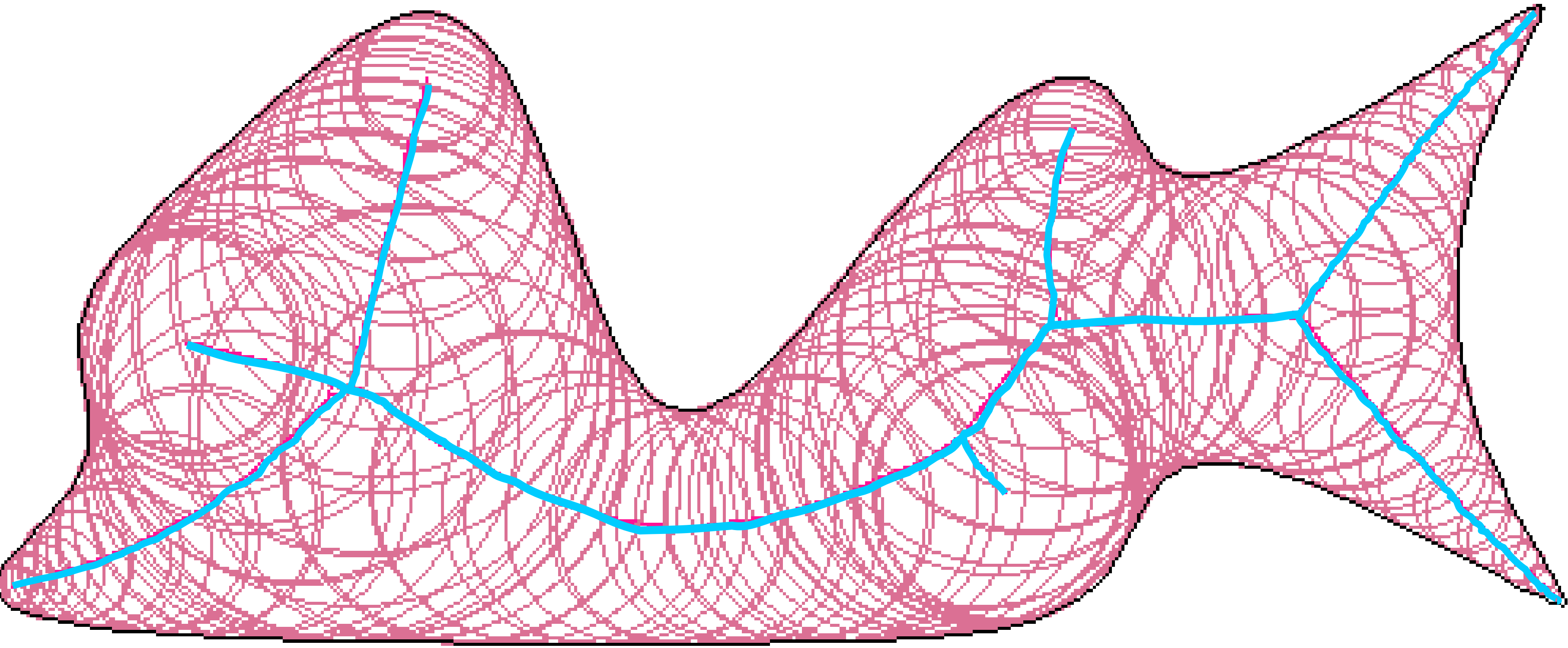
Skeleton

- *result of the Medial Axis Transform:* object points having at least two closest boundary points;
- *praire-fire analogy:* the boundary is set on fire and skeleton is formed by the loci where the fire fronts meet and quench each other;
- the locus of the centers of all the maximal inscribed hyper-spheres.

Nearest boundary points and inscribed hyper-spheres

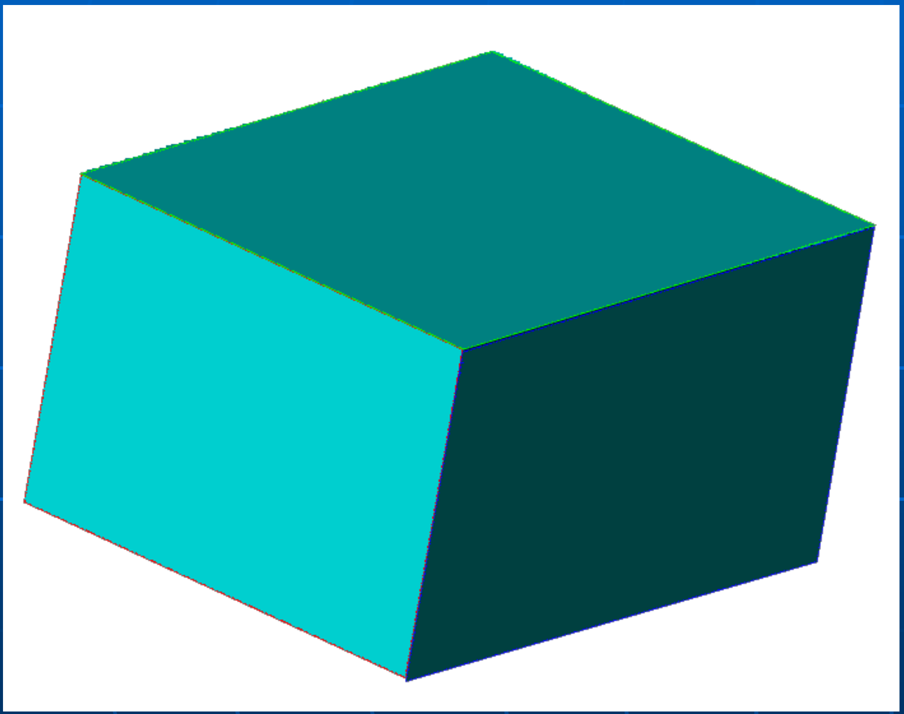


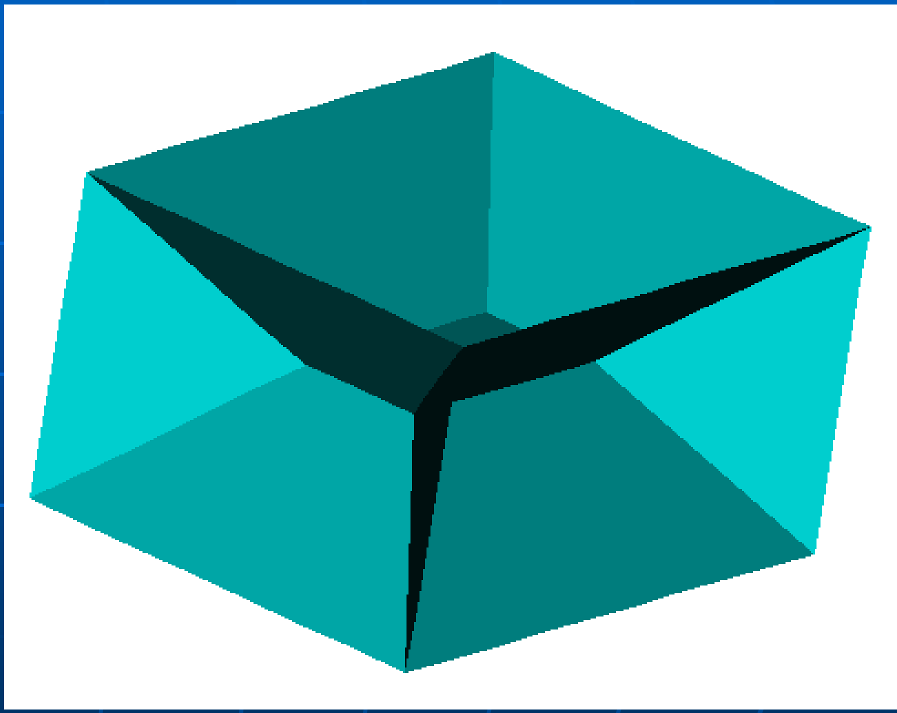
Object = union of the inscribed hyper-spheres



object boundary, maximal inscribed disks and their centers

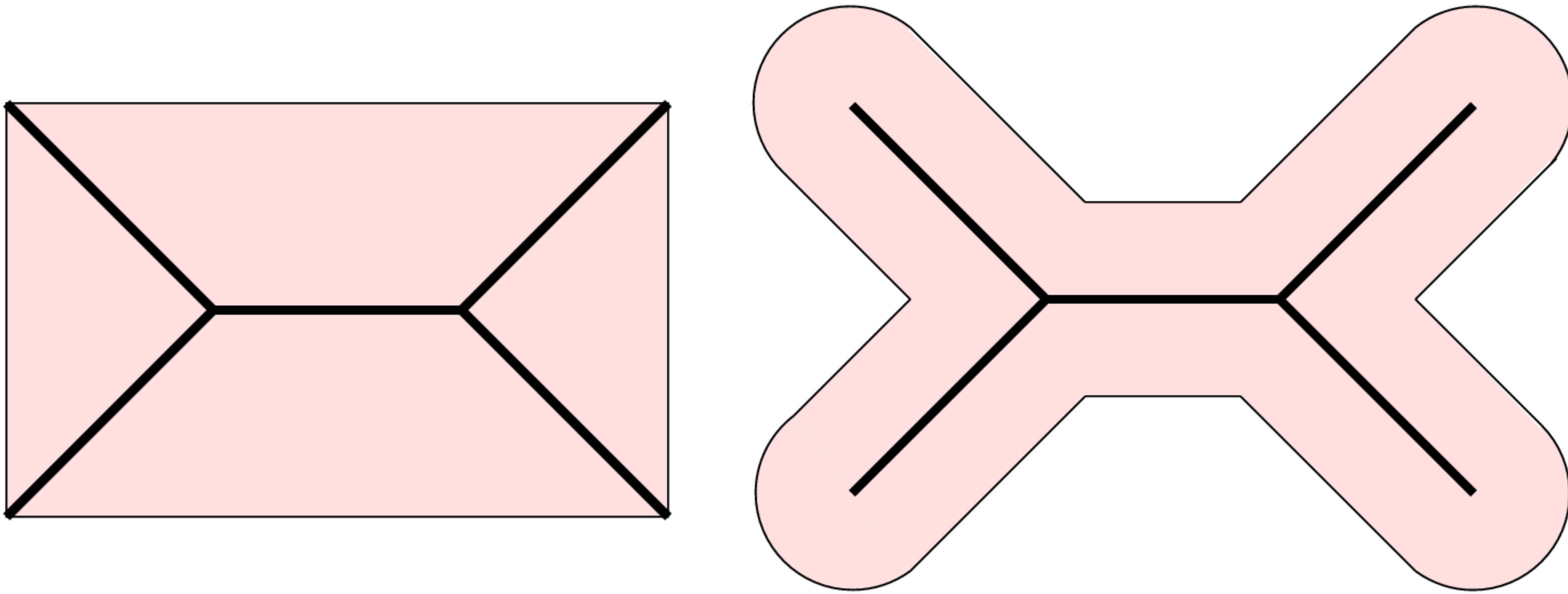
Skeleton in 3D





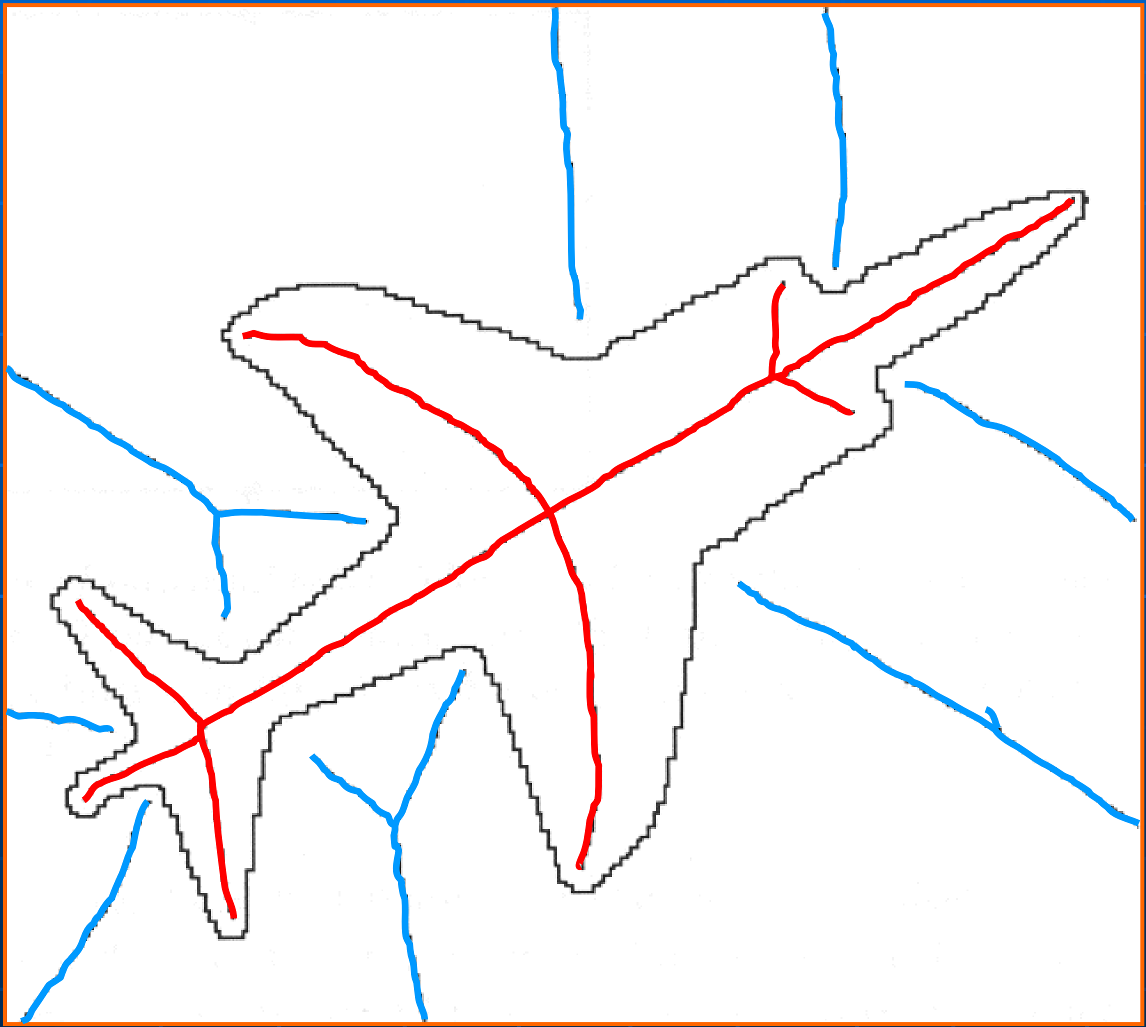
The skeleton in 3D generally contains surface patches (2D segments).

Uniqueness



The same skeleton may belong to different elongated objects.

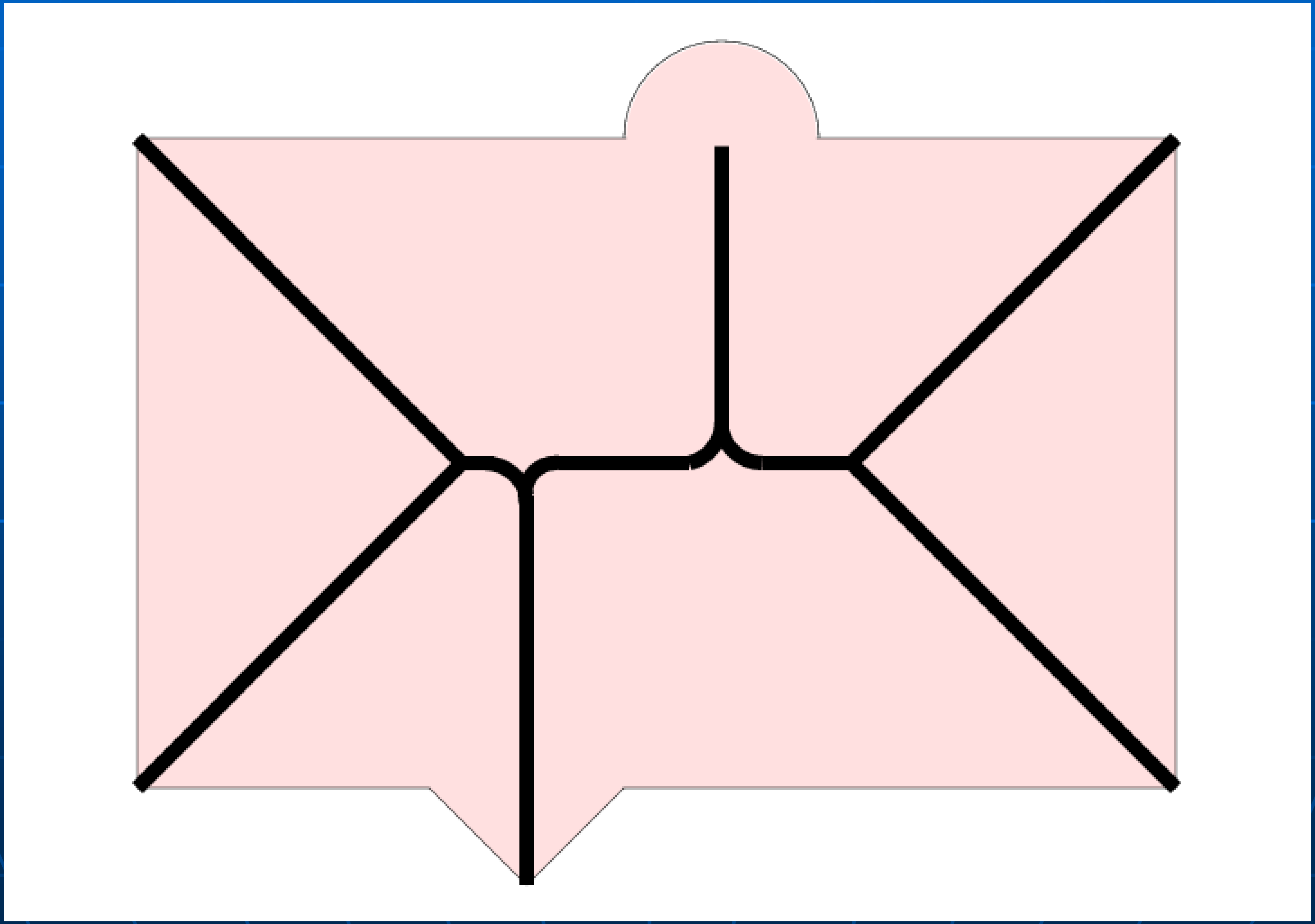
Inner and outer skeleton



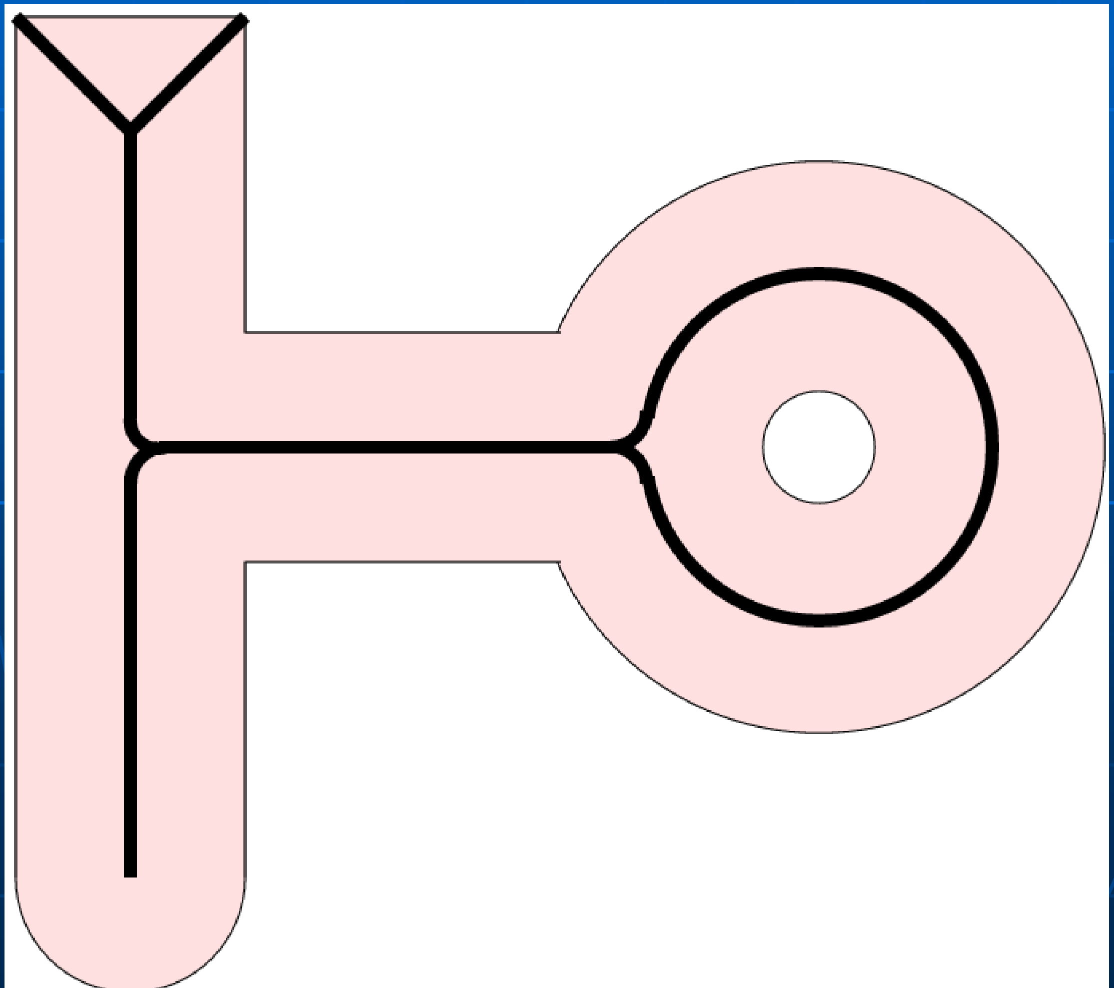
(inner) skeleton

outer skeleton
(skeleton of the negative image)

Stability



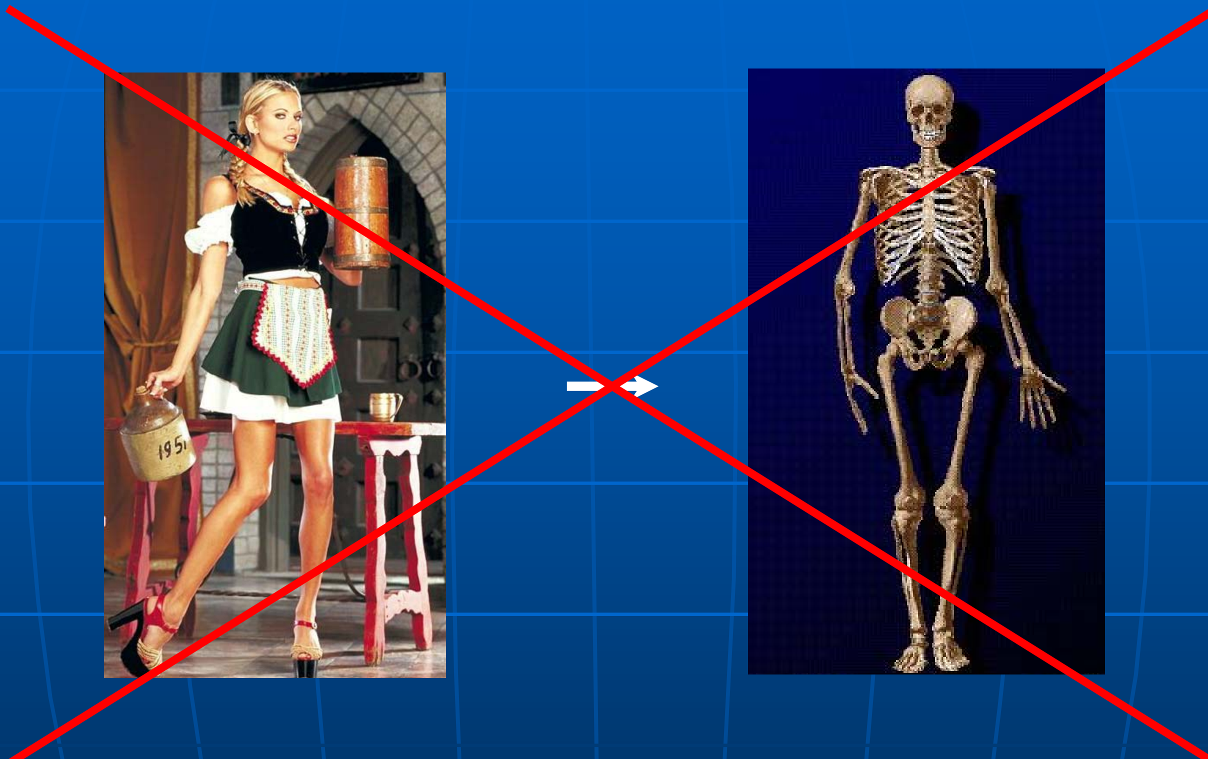
Representing the topological structure



Properties

- represents
  - the general form of an object,
  - the topological structure of an object, and
  - local object symmetries.
- invariant to
  - translation,
  - rotation, and
  - (uniform) scale change.
- simplified and thin.

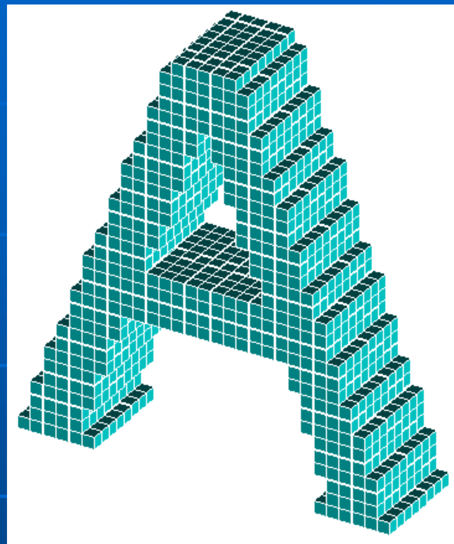
Skeletonization …

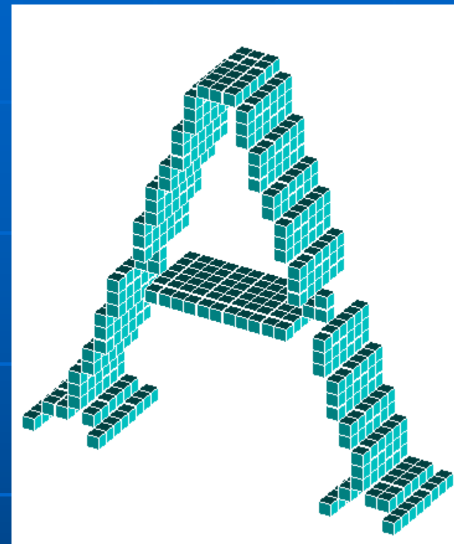


… means skeleton extraction from elongated binary objects.

Skeleton-like descriptors in 3D

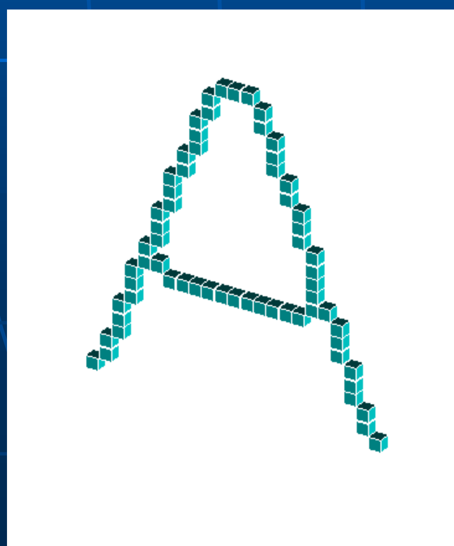
original

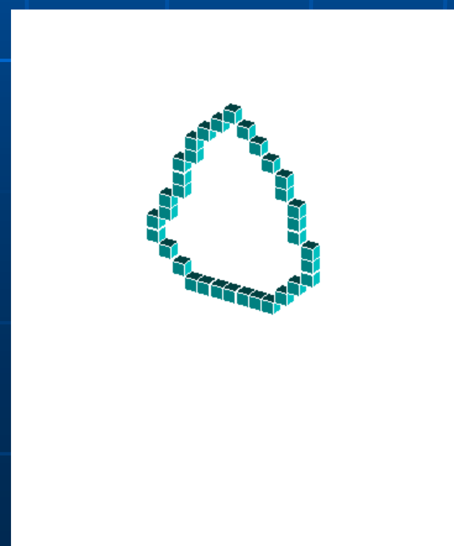




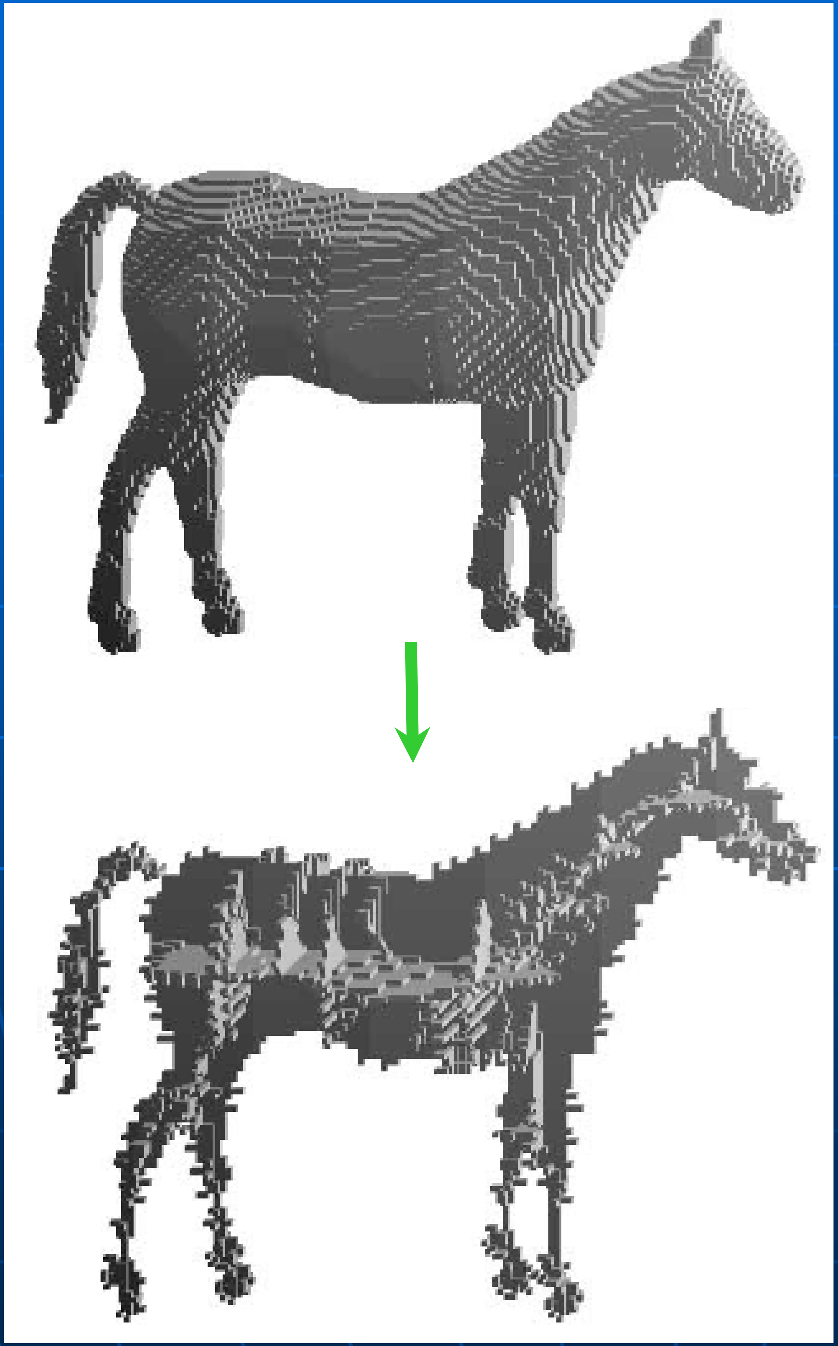
medial
surface

medial
lines



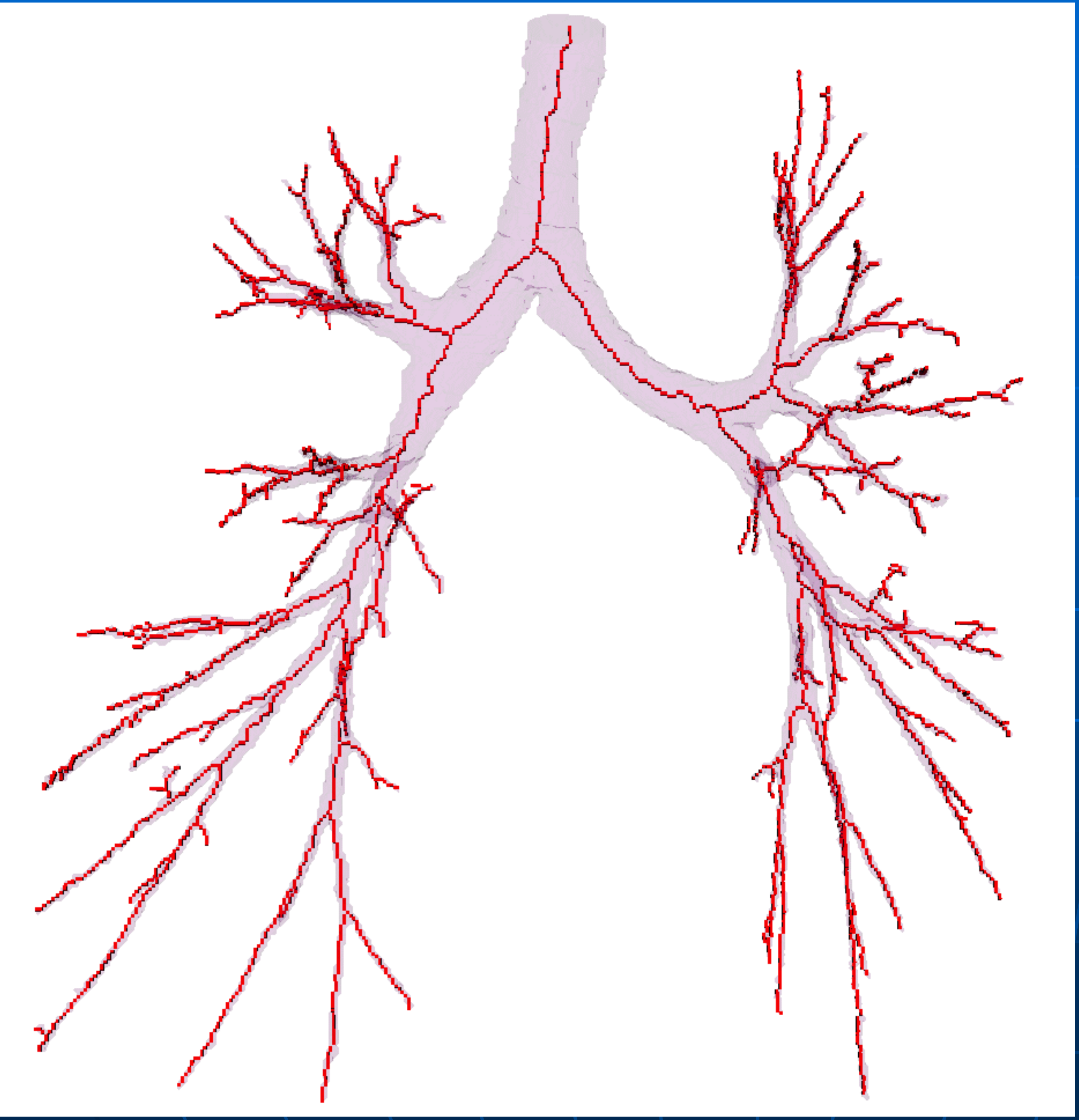


topological
kernel



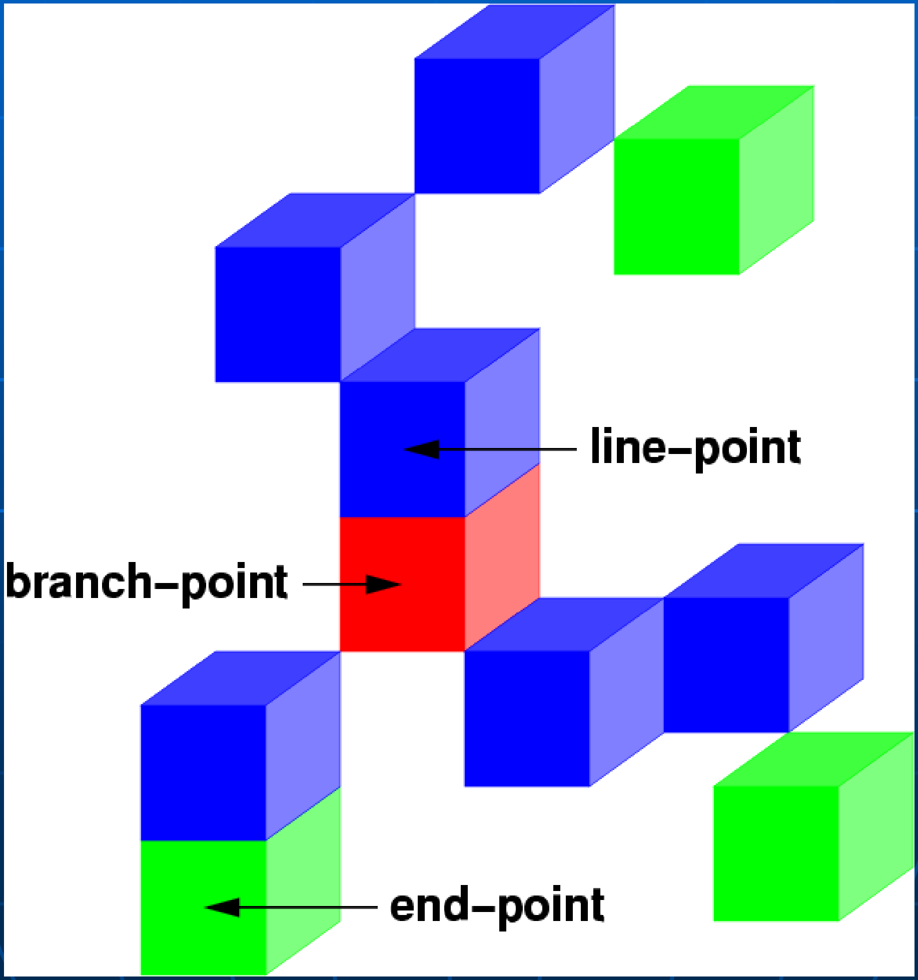
Example of medial surface

S. Svensson (SUAS, Uppsala

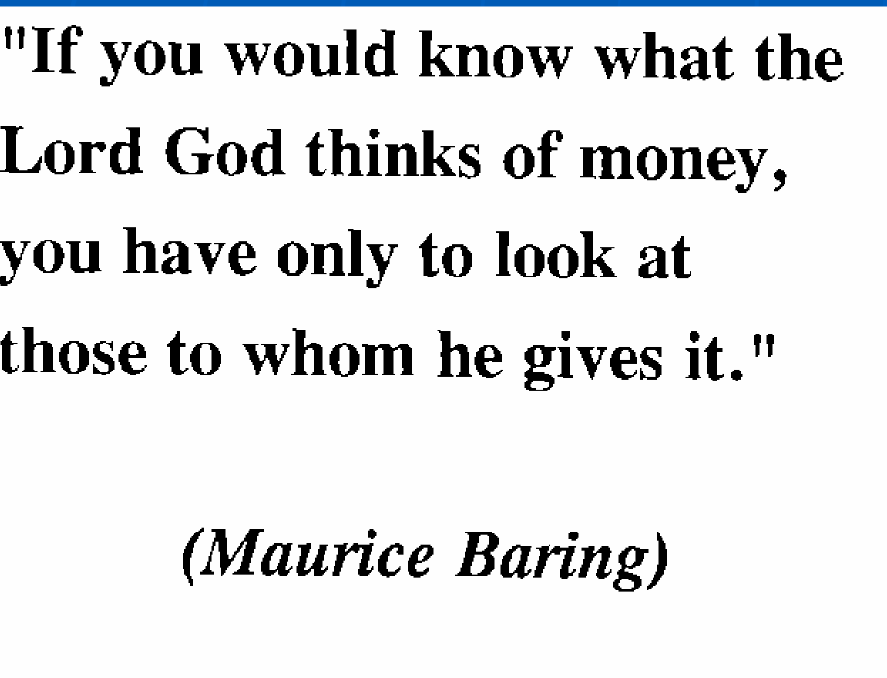


Example of medial lines

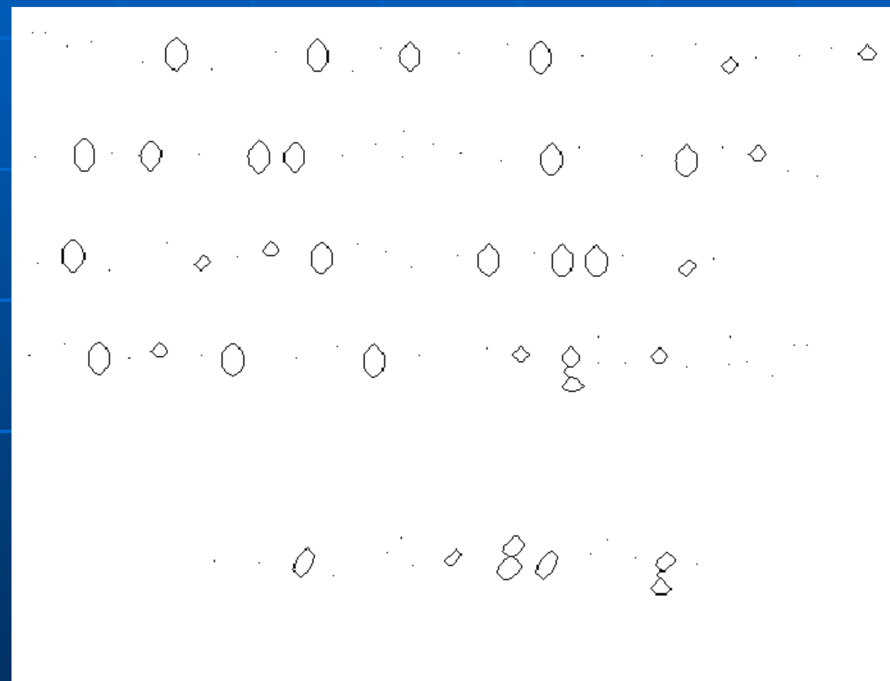
Skeletal points in 2D – points in 3D centerlines



Example of topological kernel

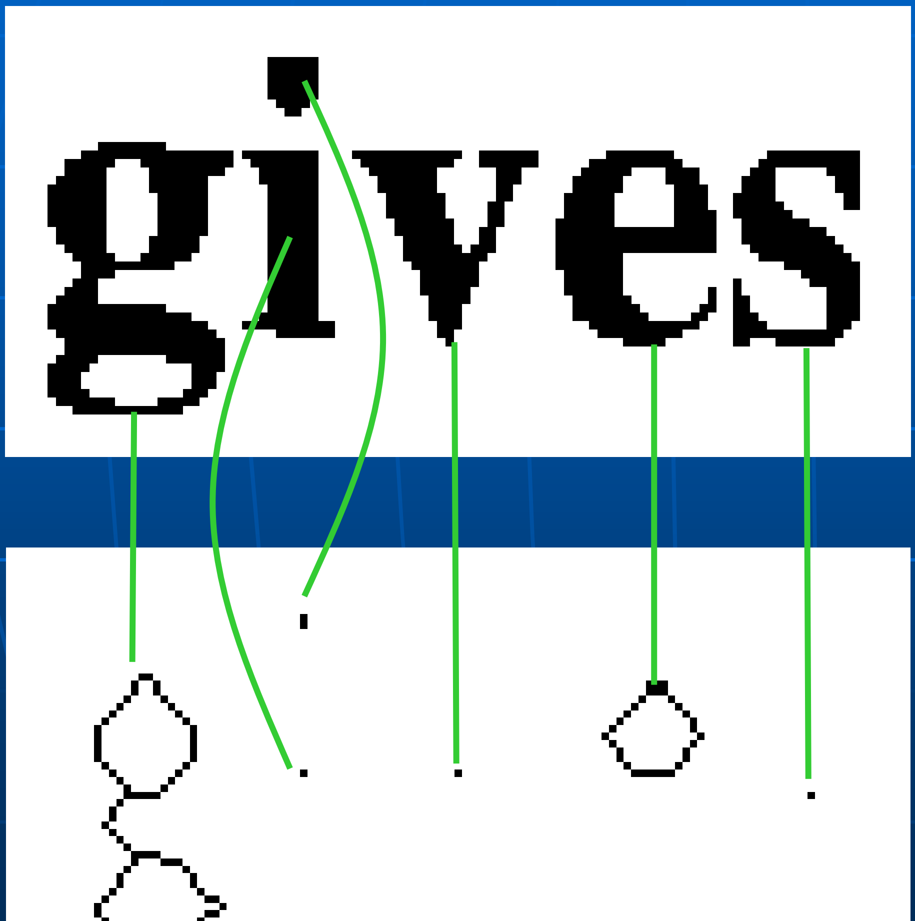


original image



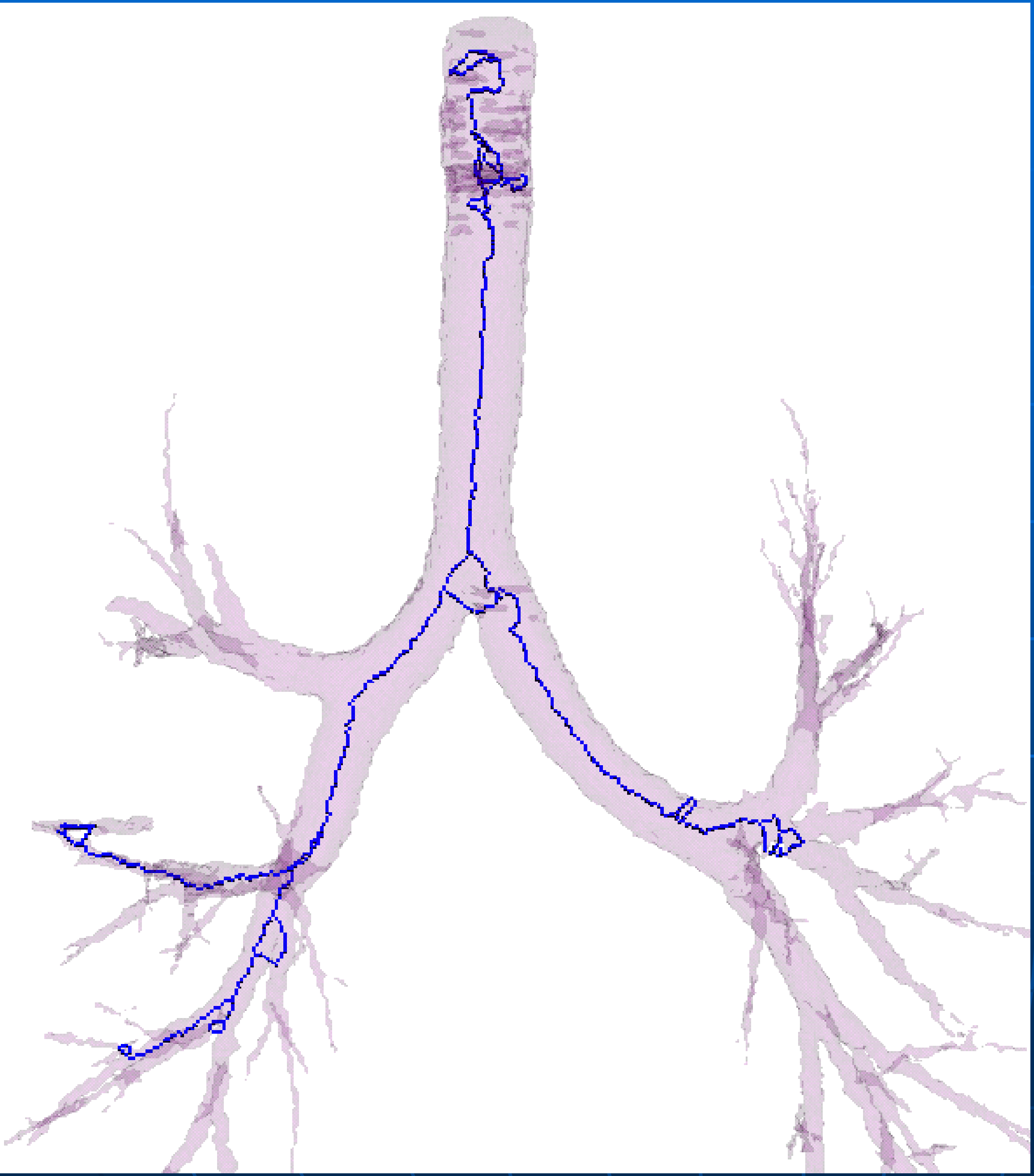
topological kernel

Example of topological kernel



simply connected → an isolated point

multiply connected → closed curve



Example of topological kernel

Skeletonization techniques

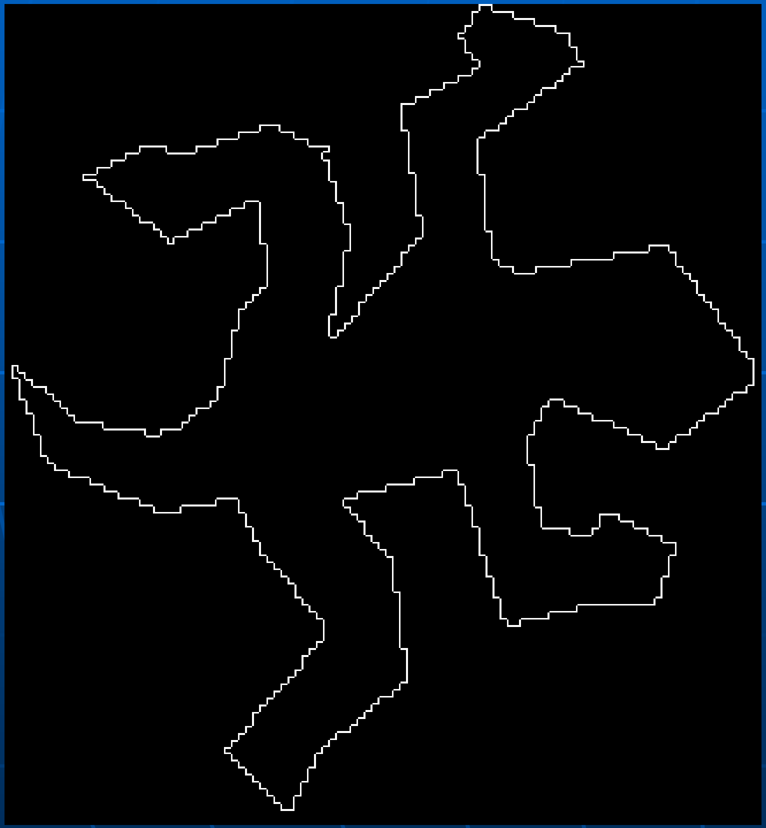
- distance transform
- Voronoi diagram
- thinning

Distance transform

Input:
Binary array *A* containing feature elements (1’s) and non-feature elements (0’s).

Output:
Non-binary array *B* containing the distance to the closest feature element.

Distance transform



input (binary)

output (non-binary)

Distance transform using city-block (or 4) distance

4	3	2	1	2	3	4
3	2	1	0	1	2	3
2	1	0	1	0	1	2
2	1	0	1	1	0	1
1	0	1	2	2	1	0
1	0	1	2	3	2	1
0	1	2	3	4	3	2

	1	
1	0	1
	1	

Distance transform using chess-board (or 8) distance

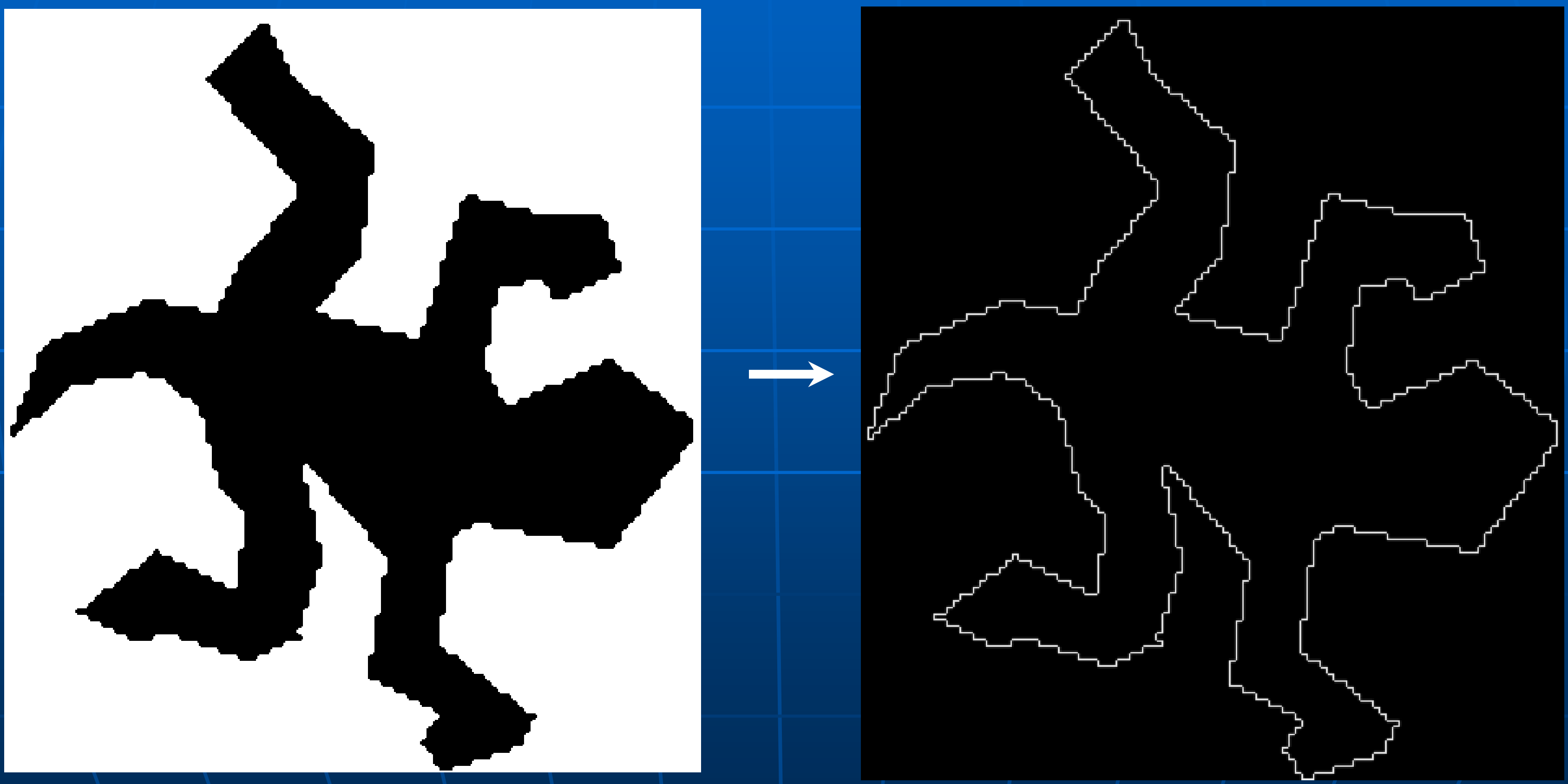
2	2	1	1	1	2	2
2	1	1	0	1	1	2
2	1	0	1	0	1	1
1	1	0	1	1	0	1
1	0	1	1	1	1	0
1	0	1	2	2	1	1
0	1	1	2	2	2	2

1	1	1
1	0	1
1	1	1

Distance-based skeletonization

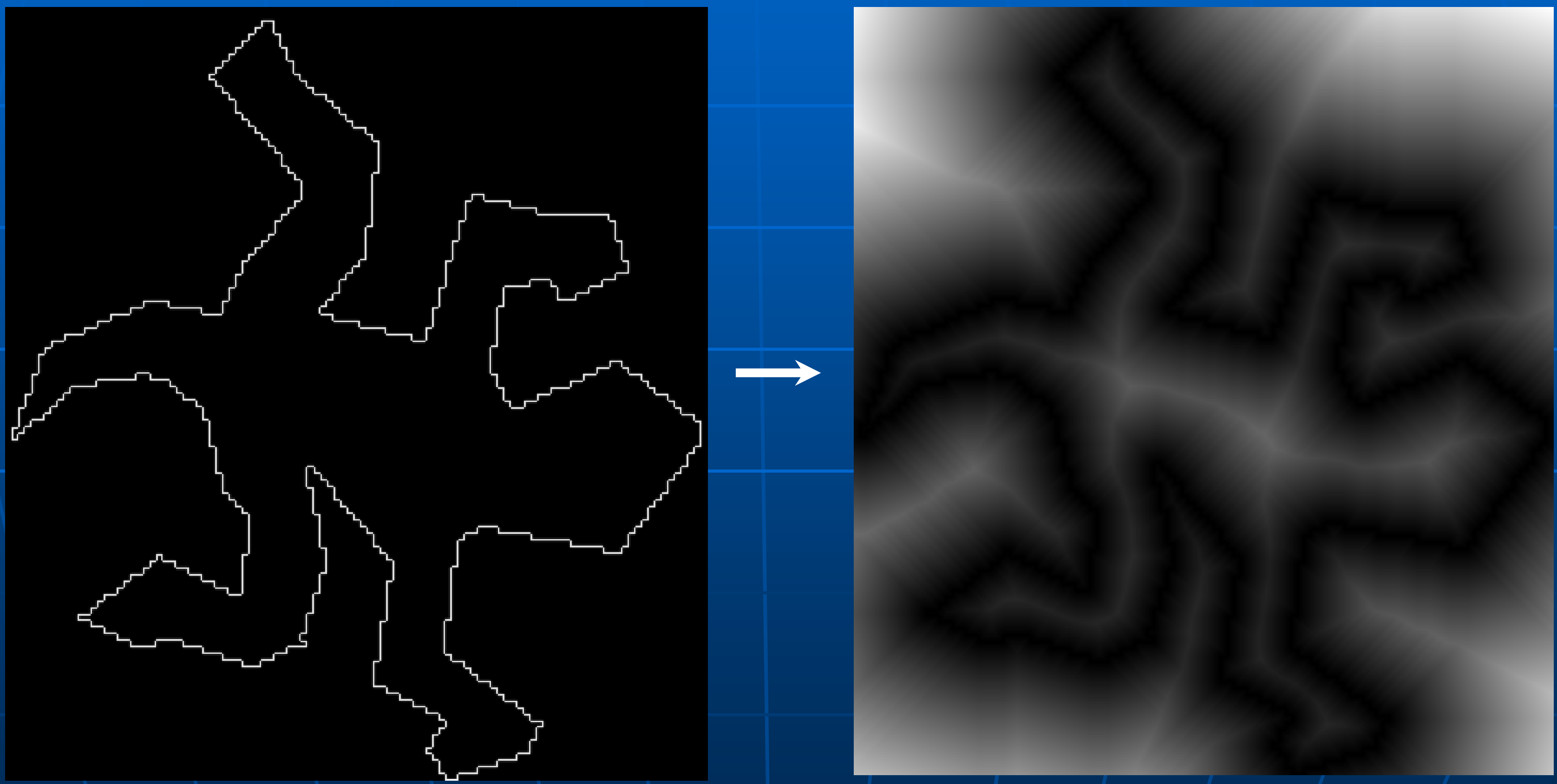
1. Border points (as feature elements) are extracted from the original binary image.
2. Distance transform is executed (i.e., distance map is generated).
3. The ridges (local extremas) are detected as skeletal points.

Distance-based skeletonization – step 1



Detecting border points

Distance-based skeletonization – step 2



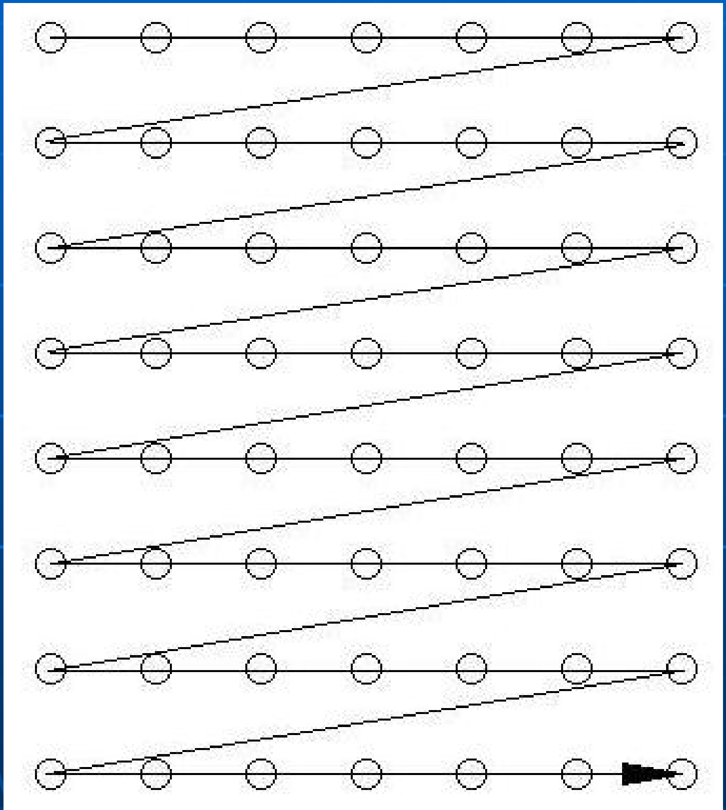
Distance mapping

Linear-time distance mapping

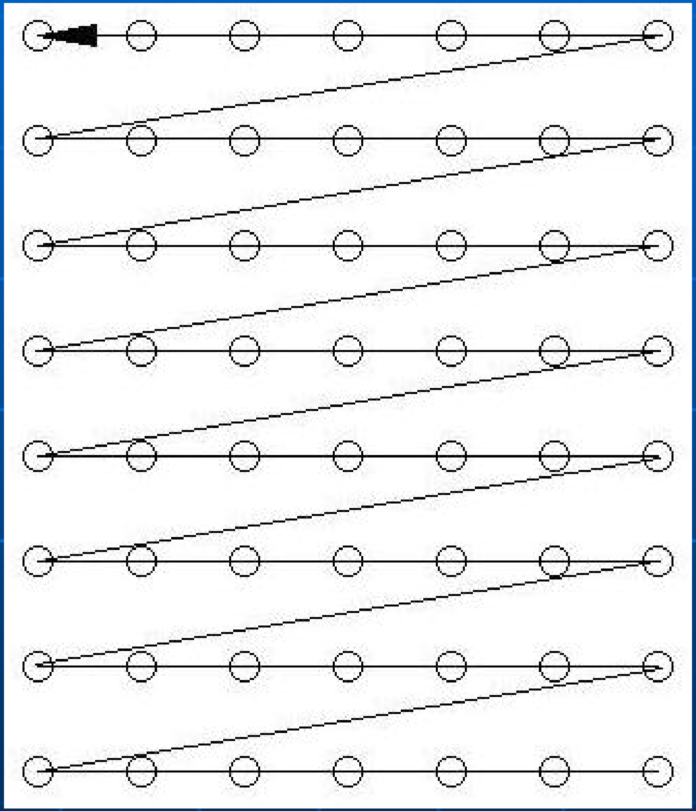
(G. Borgefors, 1984)

```
remark initialization
 for i=1 to n1 do
  for j=1 to n2 do
   if a(i,j)=1 then b(i,j)=0
   else             b(i,j)=∞
remark forward scan
 for i=1 to n1 do
  for j=1 to n2 do
   b(i,j)=min{
              b(i-1,j-1)+d2,
              b(i-1,j  )+d1,
              b(i-1,j+1)+d2,
              b(i  ,j-1)+d1,
              b(i  ,j  )
             }
remark backward scan
 for i=n1 downto 1 do
  for j=n2 downto 1 do
   b(i,j)=min{
              b(i  ,j  )    ,
              b(i  ,j+1)+d1,
              b(i+1,j-1)+d2,
              b(i+1,j  )+d1,
              b(i+1,j+1)+d2
             }
```

Linear-time distance mapping

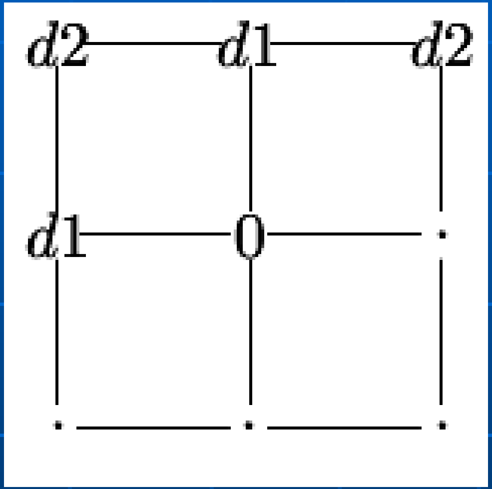


forward scan

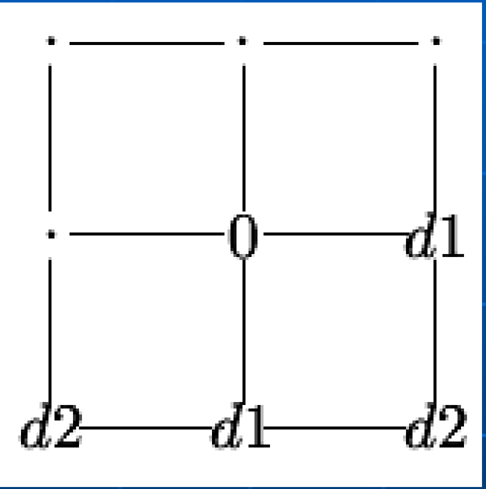


backward scan

Linear-time distance mapping



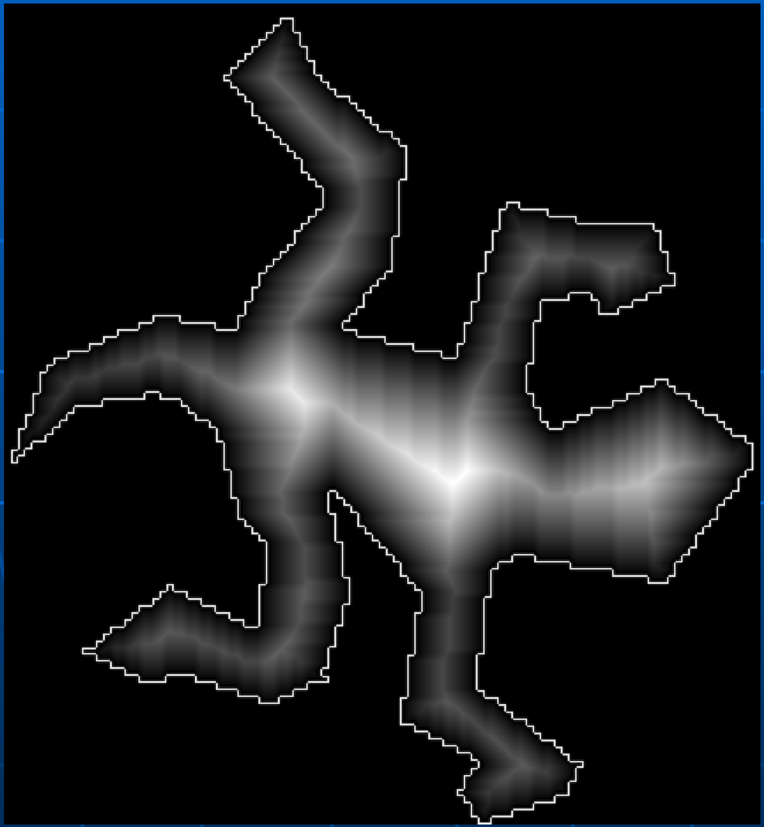
forward scan

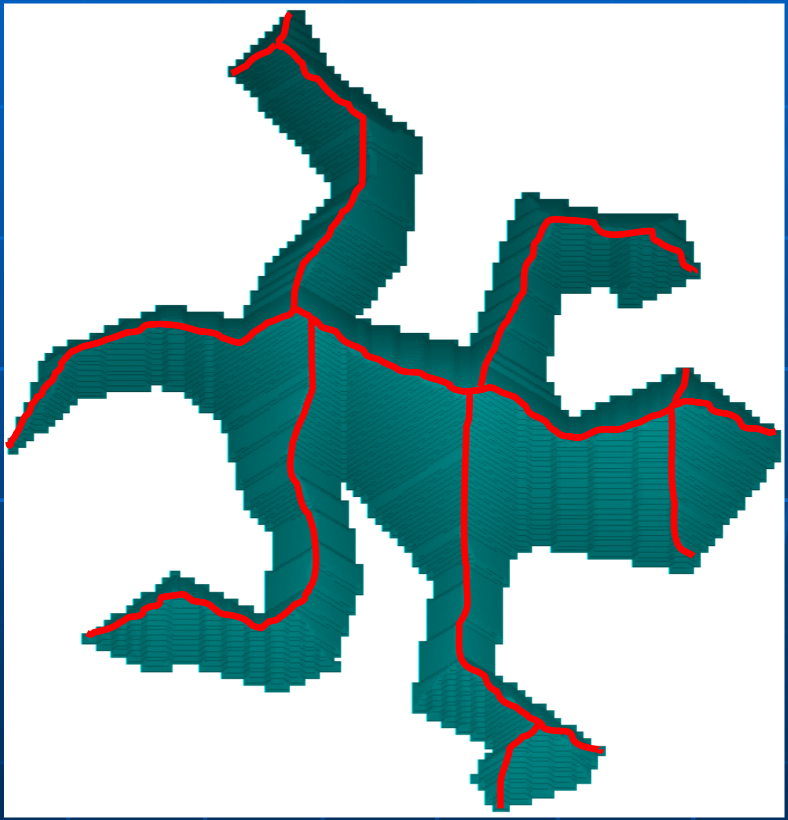


backward scan

generally: d1=3, d2=4

Distance-based skeletonization – step 3





Detecting ridges (local extremas)

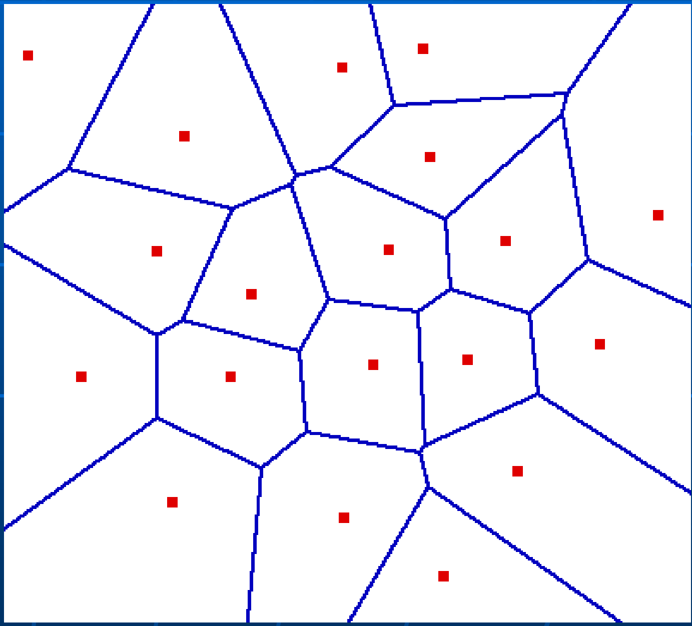
Ridge detection

$$H(f) = \begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_d} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_d} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_d \partial x_1} & \frac{\partial^2 f}{\partial x_d \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_d^2} \end{pmatrix}$$

… by analyzing the eigenvalues and eigenvectors of the negative Hessian matrix.

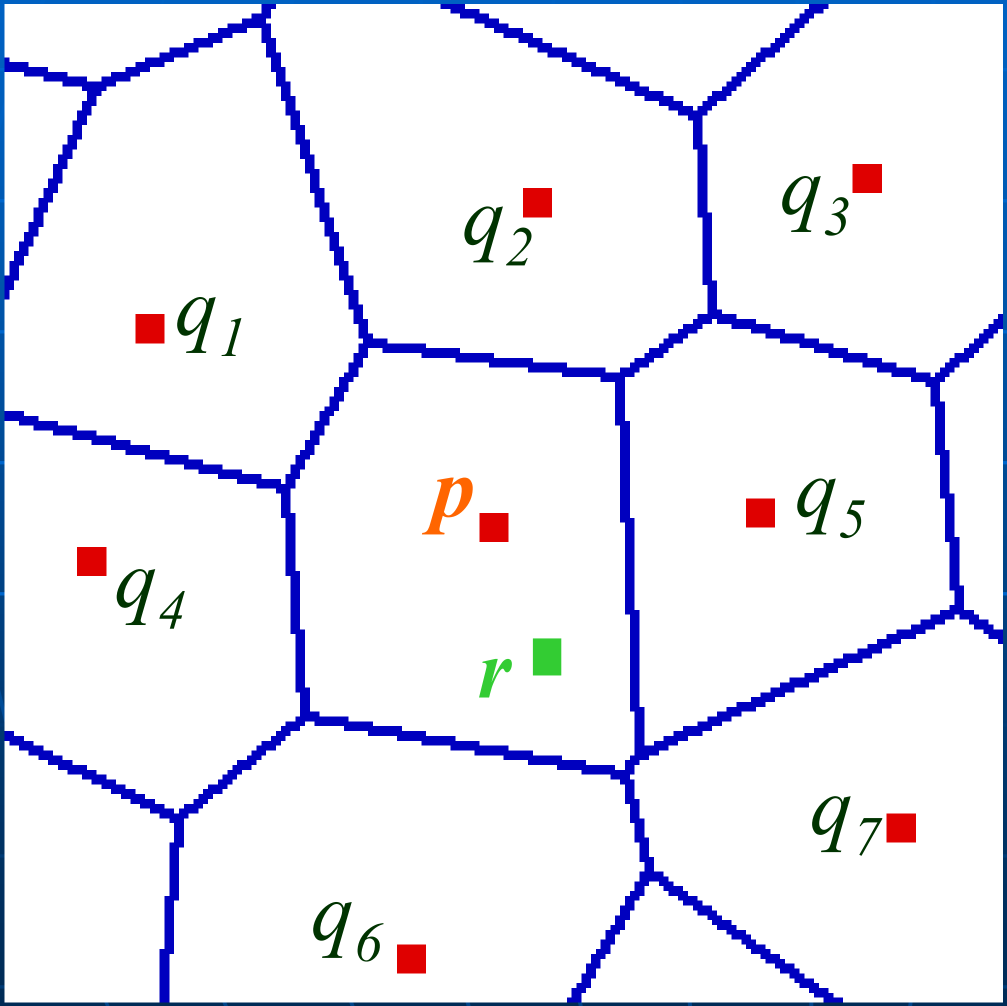
Voronoi diagram

<u>Input</u>:
Set of points (generating poins)



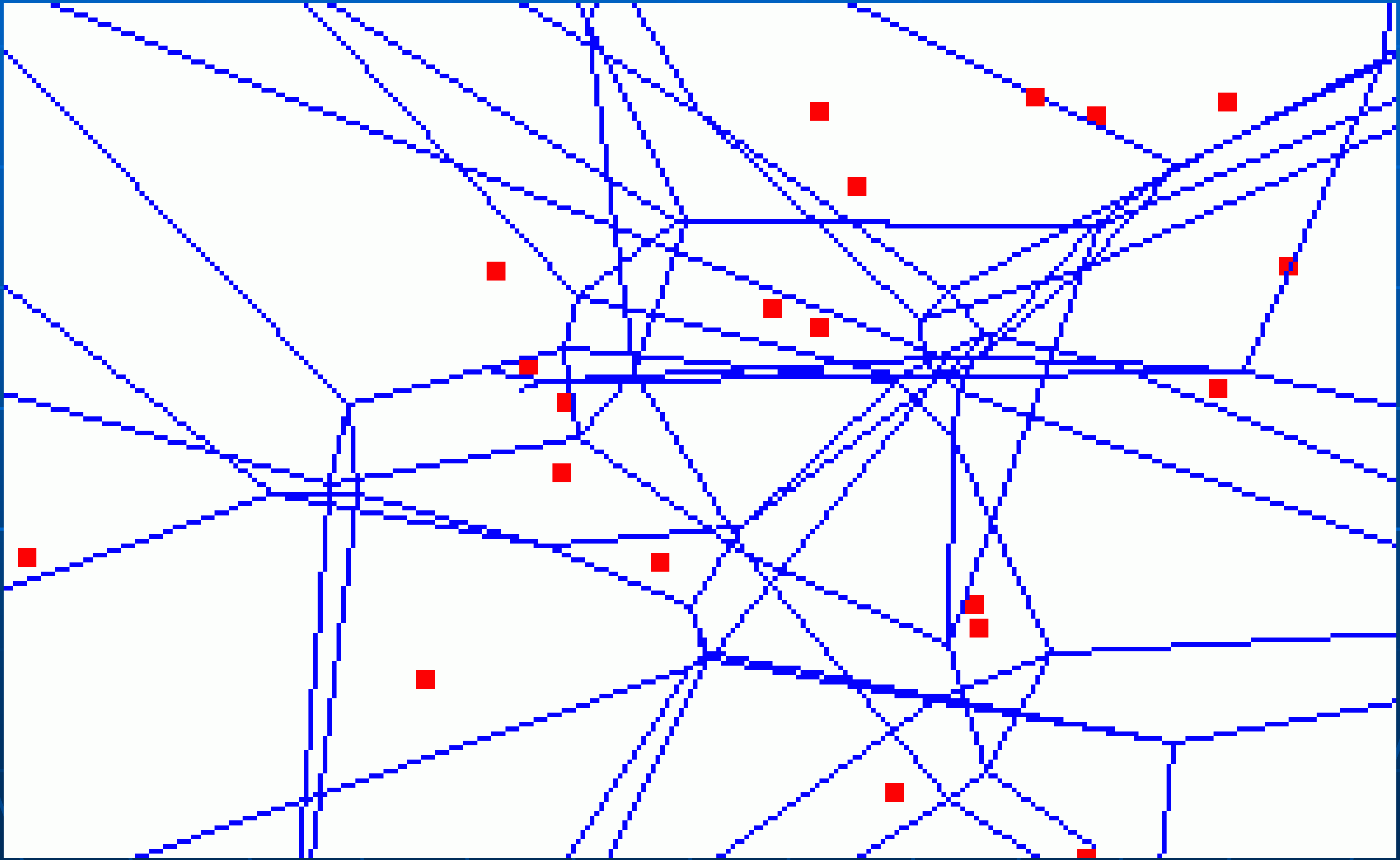
<u>Output</u>:
the partition of the space into cells so that each cell contains exactly one generating point and the locus of all points which are closer to this generating point than to others.

Voronoi diagram



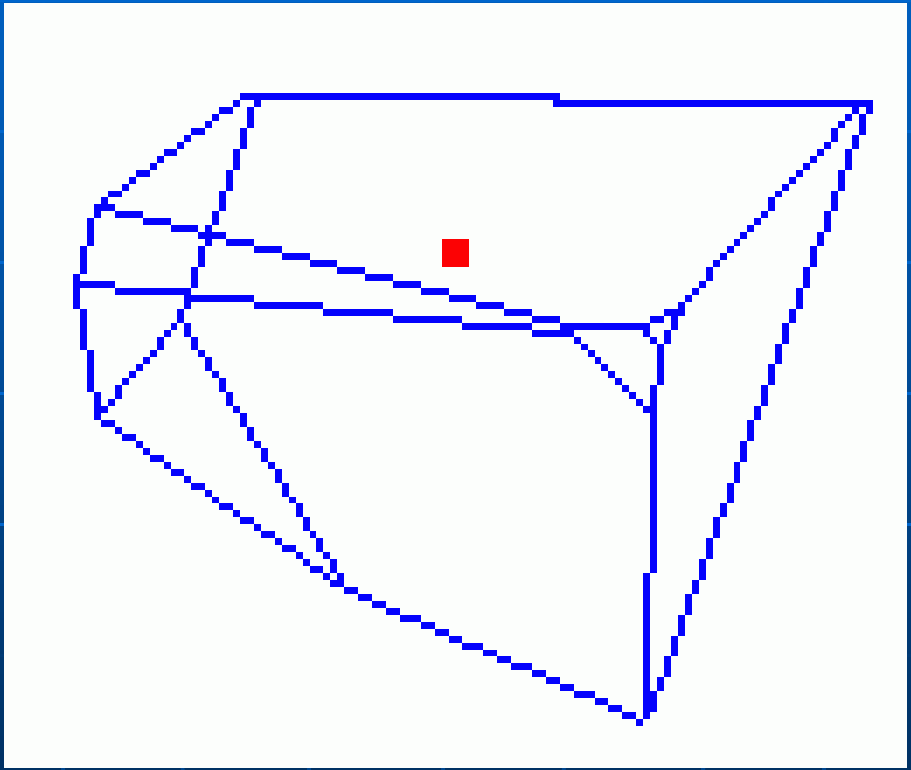
$$d(r,p) \le d(r,q_i)$$
$$(i = 1,2,\ldots)$$

Voronoi diagram in 3D



Voronoi diagram of 20 generating points

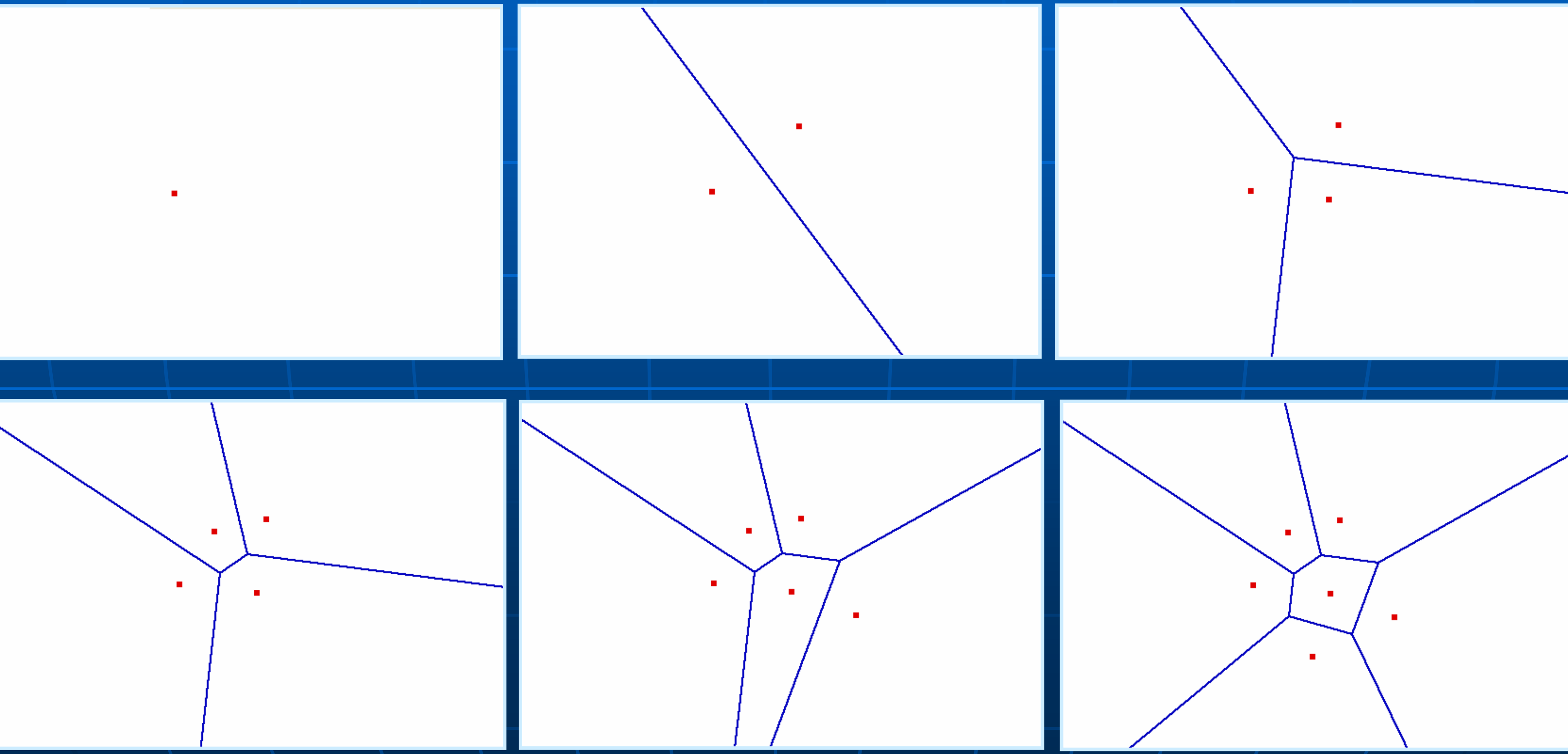
Voronoi diagram in 3D



A cell (convex polyhedron) of that Voronoi diagram

Incremental construction

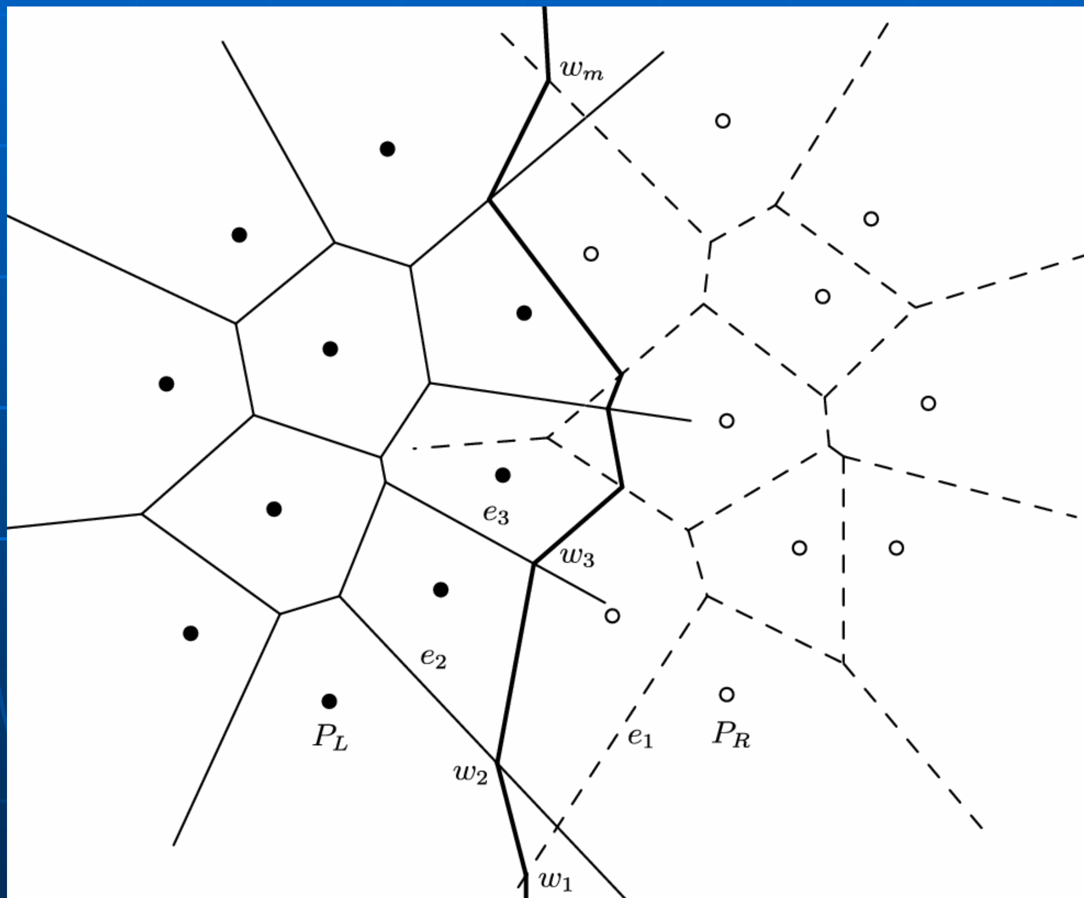
$O(n)$



Divide and conquer

O(n·log$n$)

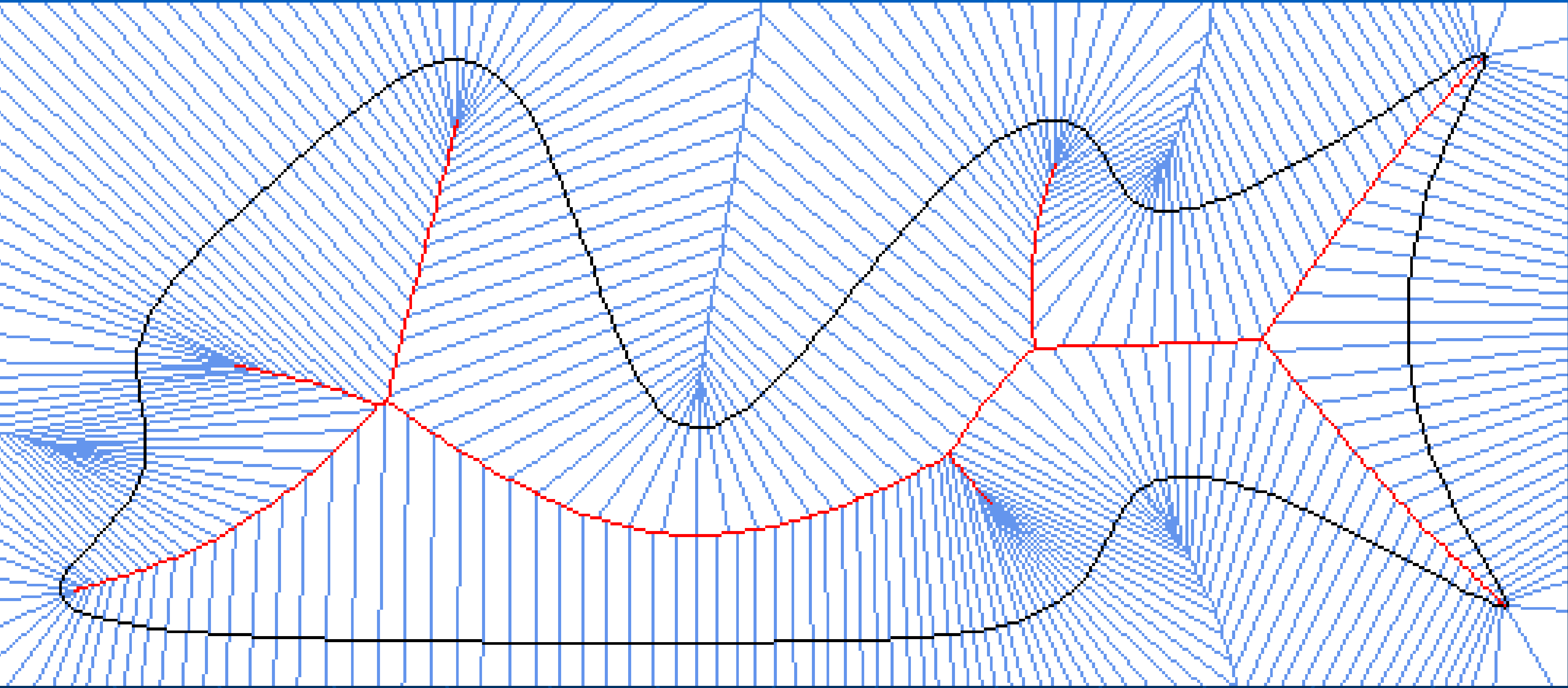
left diagram



right diagram

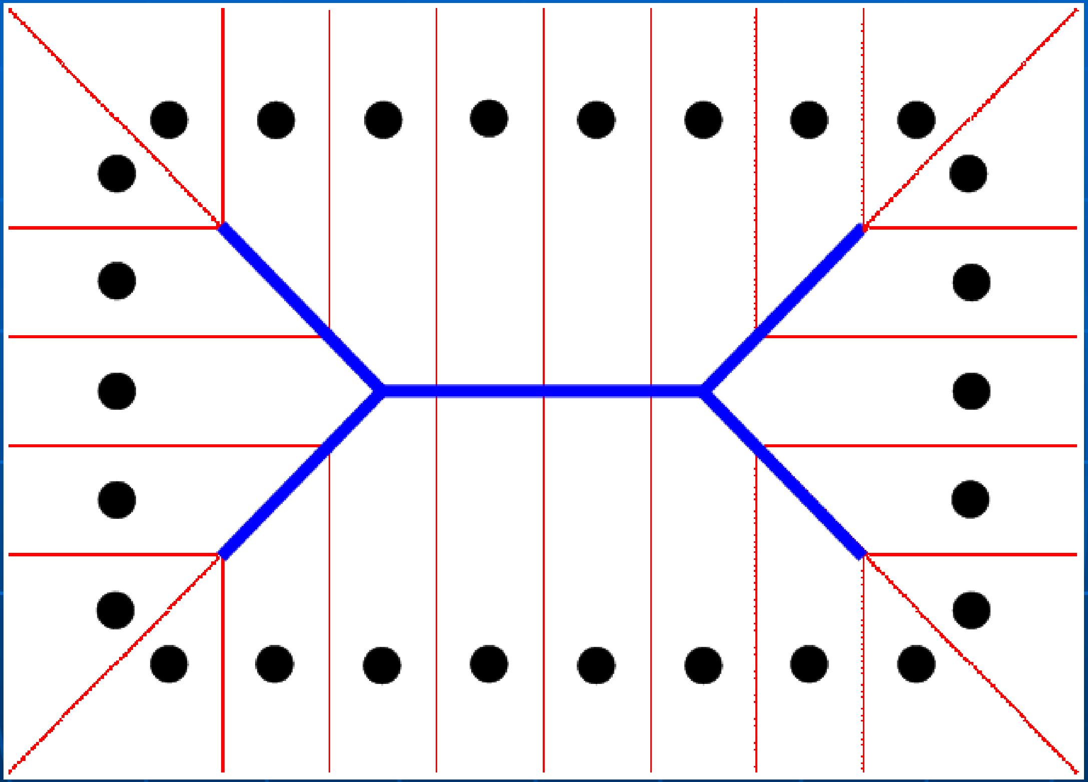
merging

Voronoi diagram - skeleton



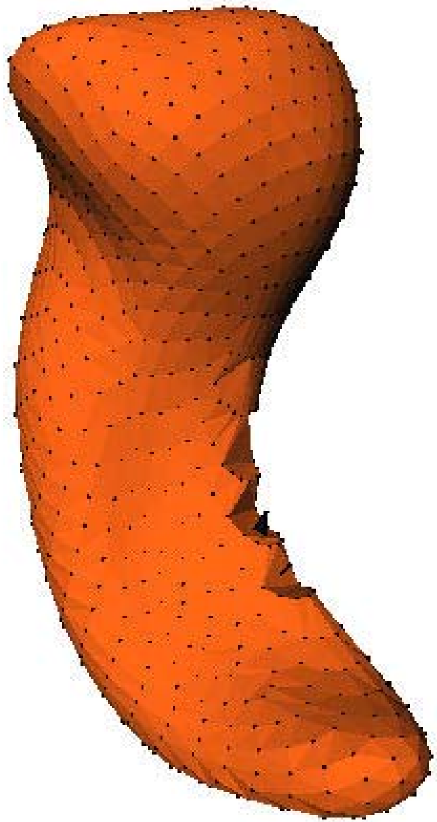
set of generating points = sampled boundary

Voronoi diagram - skeleton

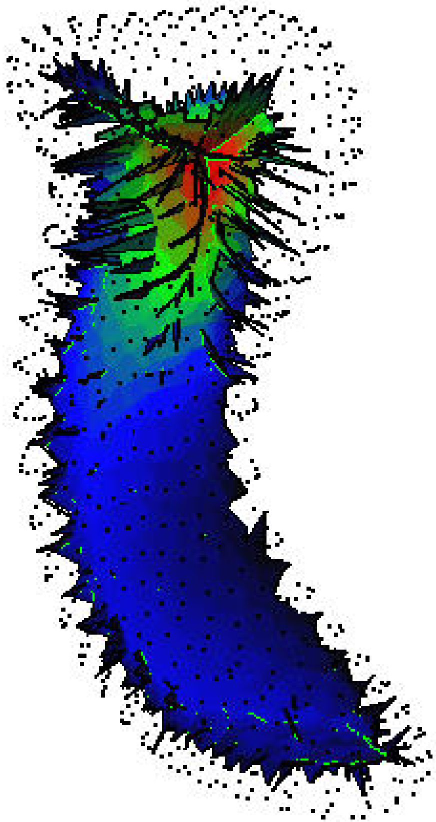


If the density of boundary points goes to infinity, then the corresponding Voronoi diagram converges to the skeleton.

Voronoi skeleton



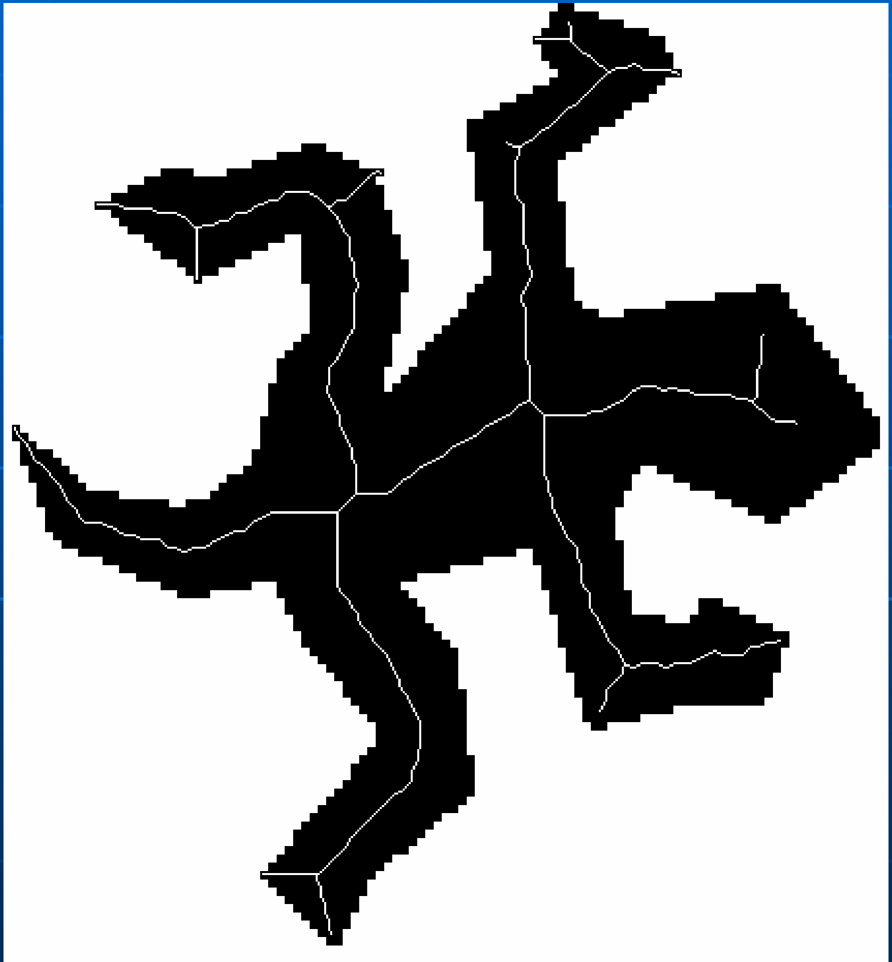
original 3D object



Voronoi skeleton

M. Styner (UNC, Chapel Hill)

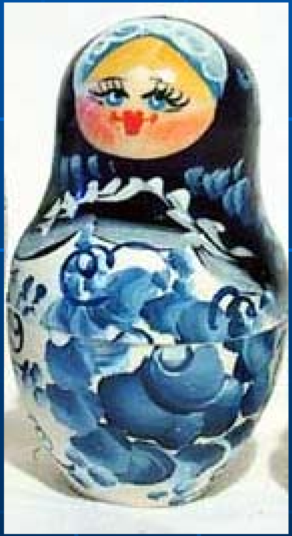
Thinning

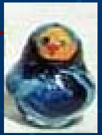


modeling fire-front propagation

Iterative object reduction

Matryoshka:
Russian nesting wooden doll.

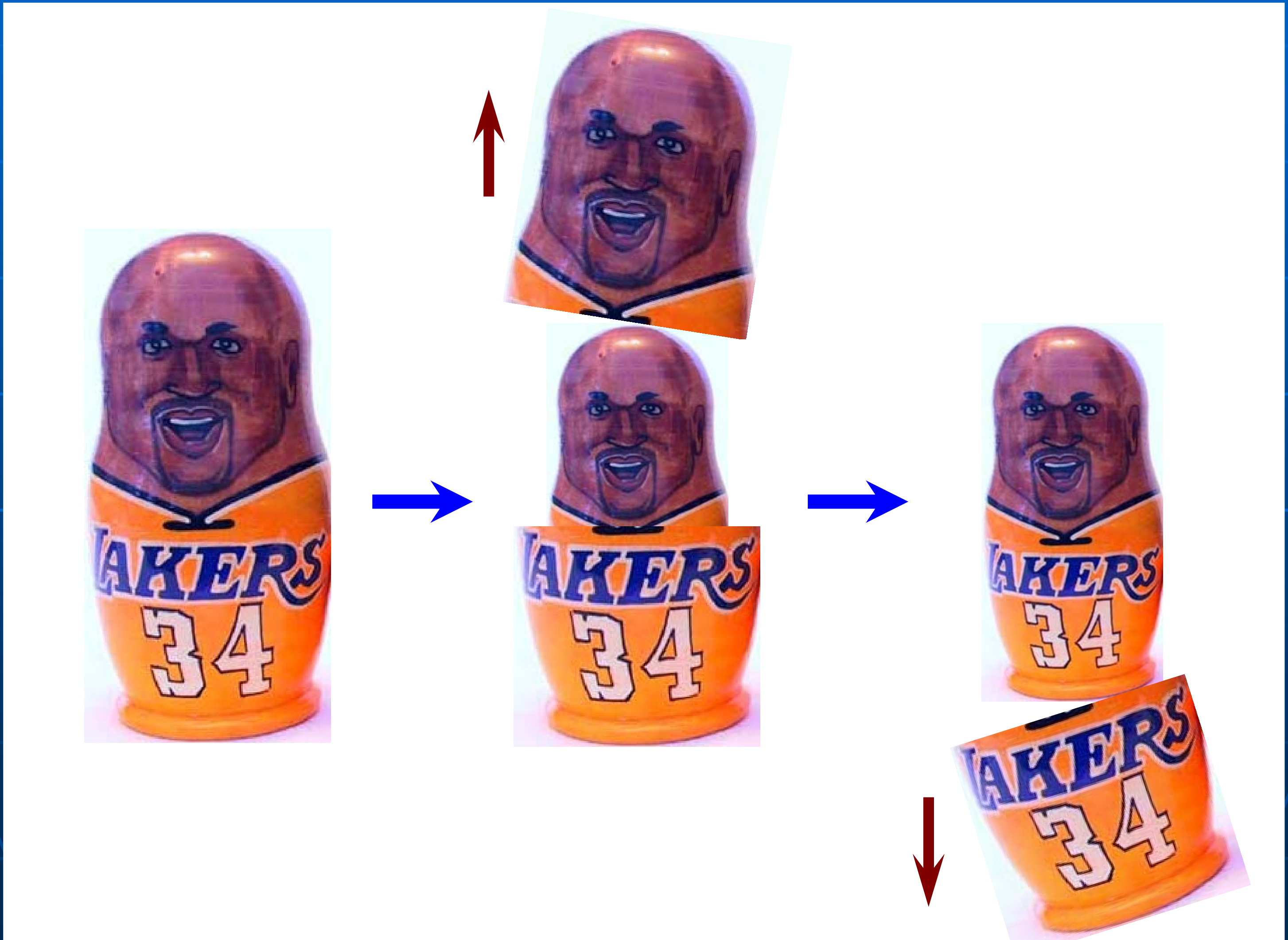




original
object

reduced
structure

One iteration step



Thinning algorithms

repeat

remove „*deletable*" border points
from the actual binary image

one
iteration
step

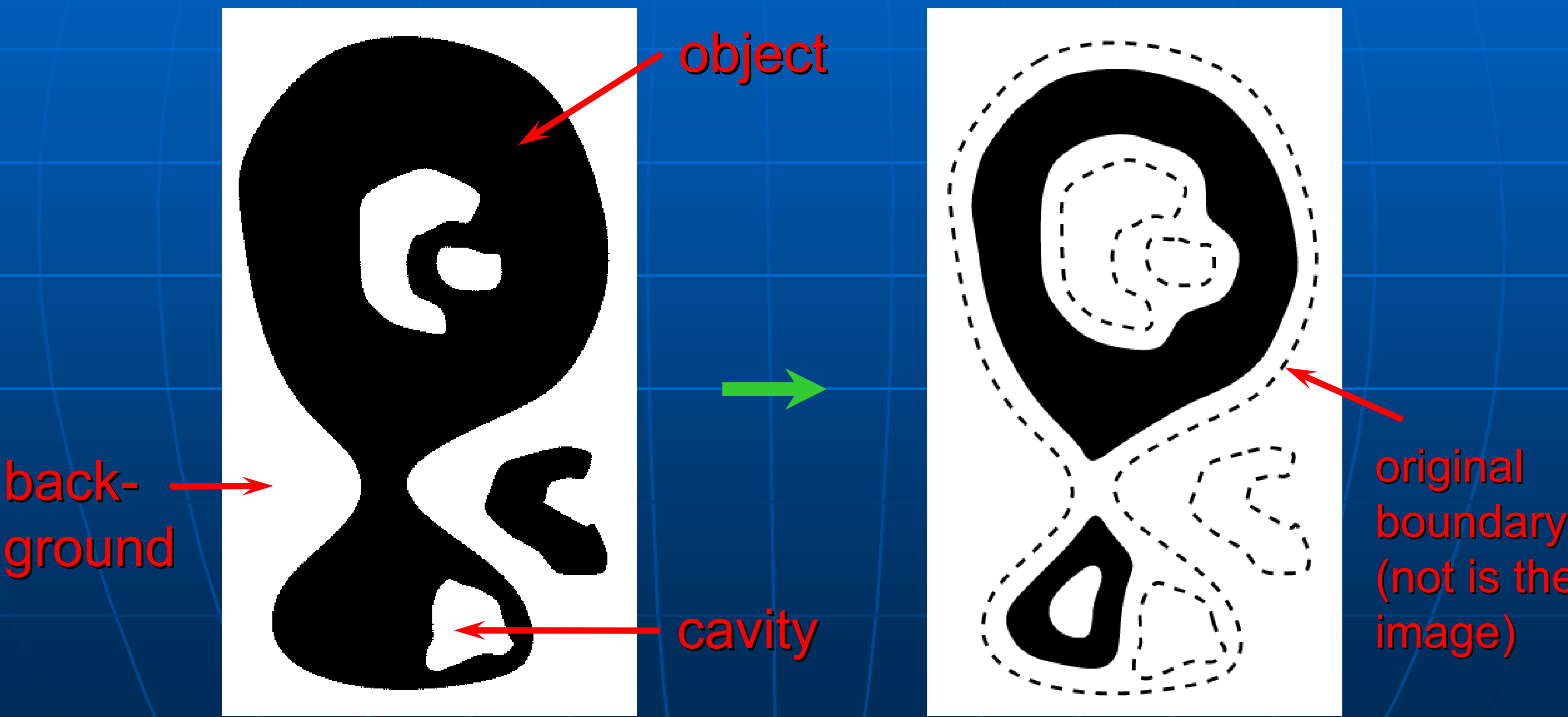
until no points are deleted

degrees of freedom:

– which points are regarded as „*deletable*" ?
– how to organize one iteration step?

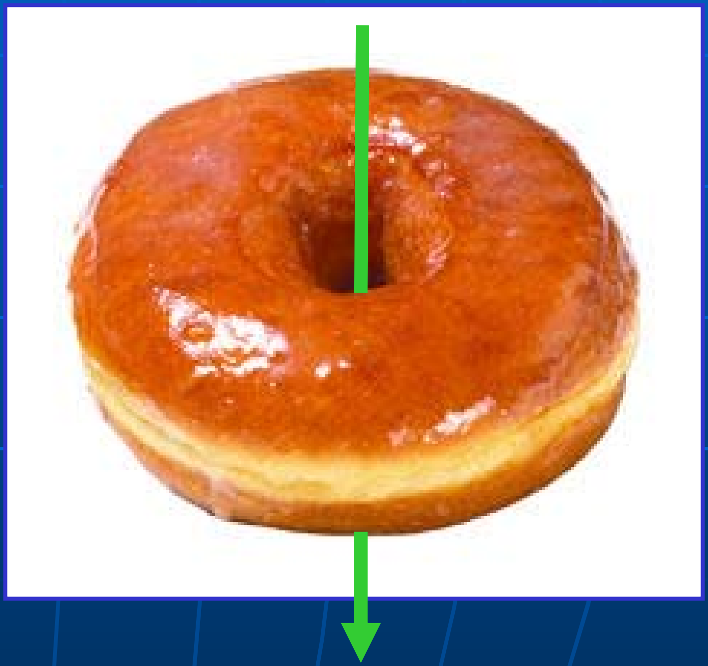
Topology preservation in 2D

(a counter example)



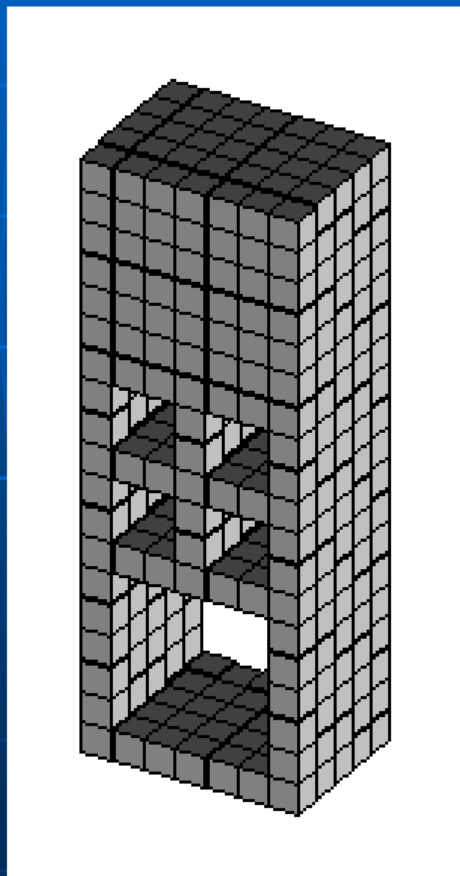
Topology in 3D

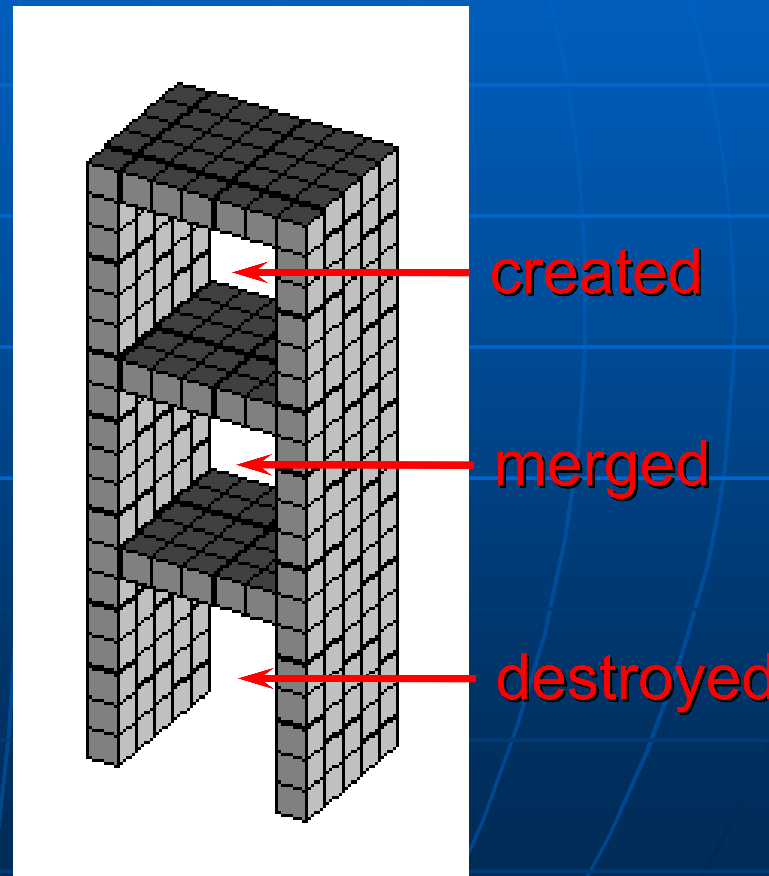
hole - a new concept



"A topologist is a man who does not know the difference between a coffee cup and a doughnut."

Topology preservation in 3D
(a counter example)





Shape preservation

"If you would know what the Lord God thinks of money, you have only to look at those to whom he gives it."

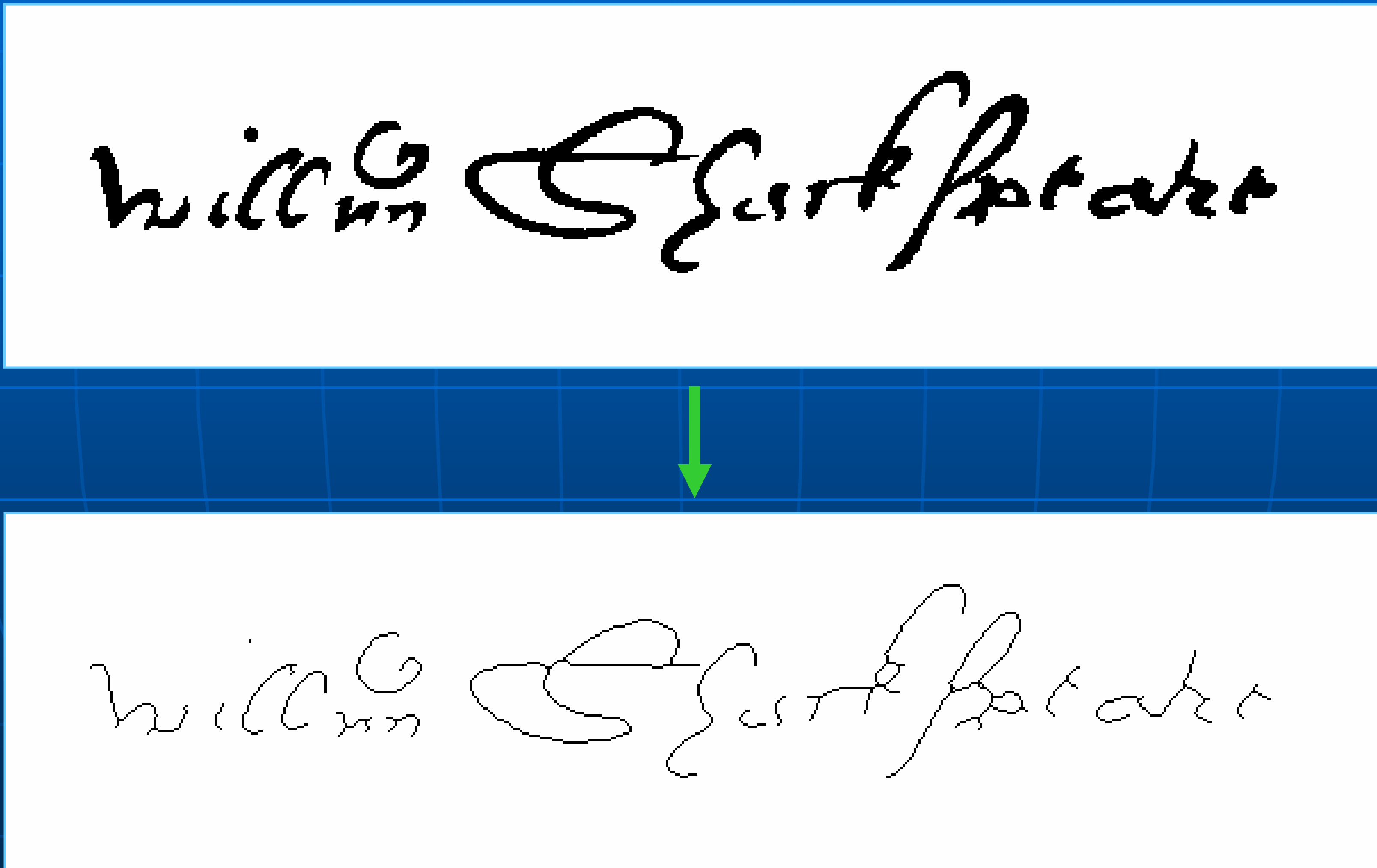
(Maurice Baring)

Shape preservation

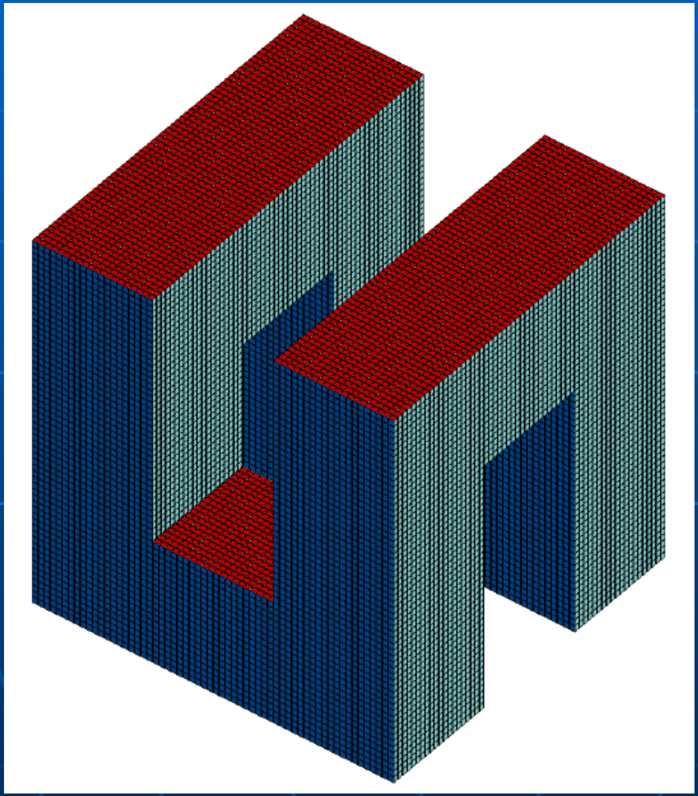
"If you would know what the Lord God thinks of money, you have only to look at those to whom he gives it."

(Maurice Baring)

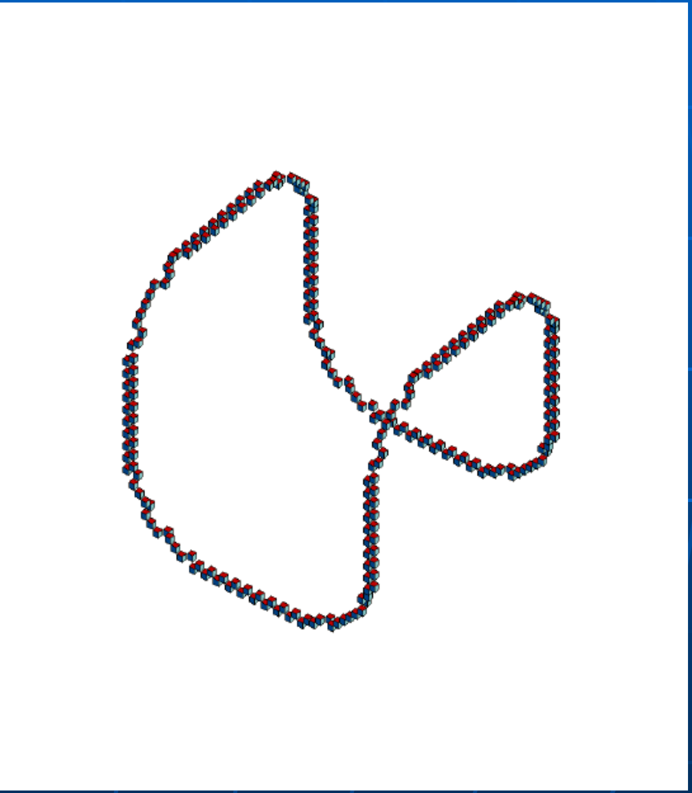
Example of 2D thining



Example of 3D thining



original object



centerline

I prefer thinning since it …

- allows direct centerline extraction in 3D,
- makes easy implementation possible,
- takes the least computational costs, and
- can be executed in parallel.

Requirements

- Geometrical:
  The skeleton must be in the middle of the original object and must be invariant to translation, rotation, and scale change.
- Topological:
  The skeleton must retain the topology of the original object.

Comparison

method	geometrical	topological
distance-based	yes	no
Voronoi-based	yes	yes
thinning	no	yes

Applications in 2D

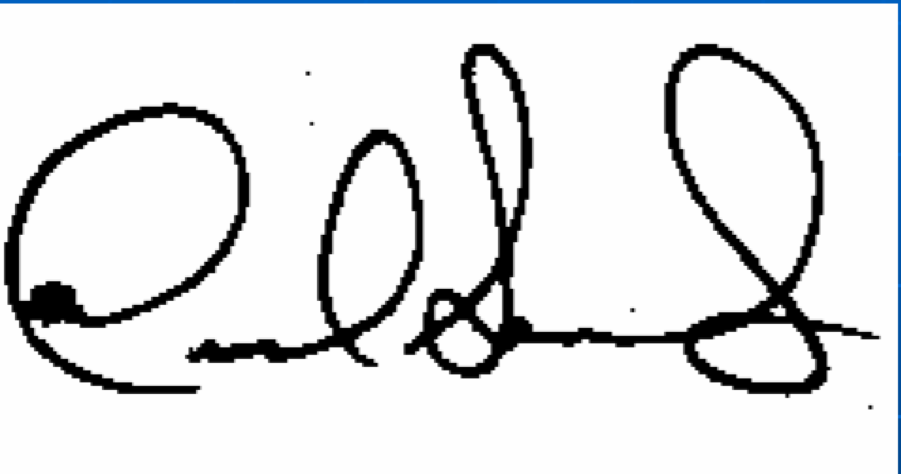
- „exotic" character recognition
- recognition of handwritten text
- signature verification
- fingerprint and palmprint recognition
- raster-to-vector-conversion
- …

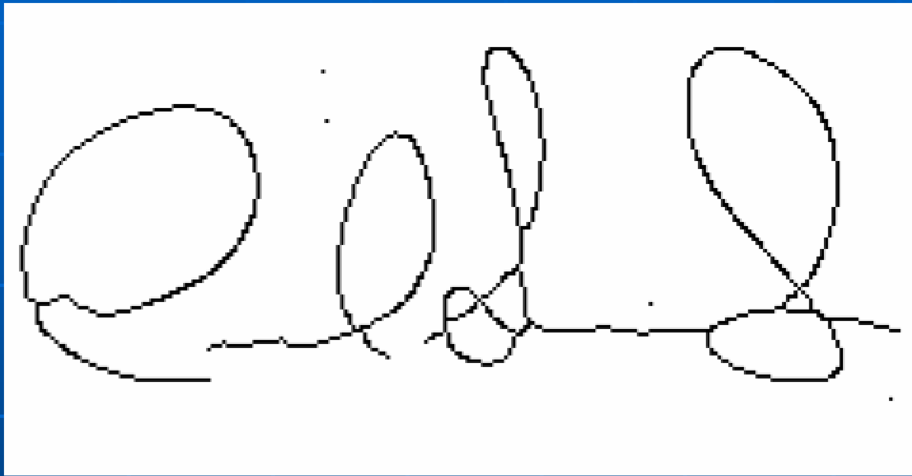
Exotic character recognition

characters of a Japanese signature

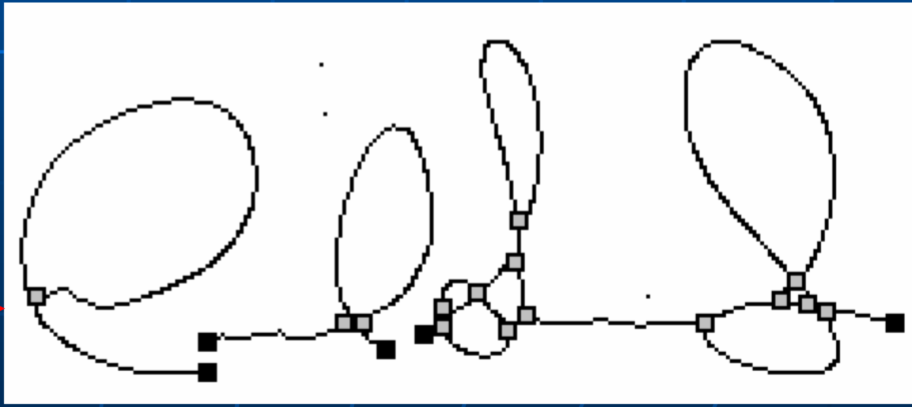
K. Ueda

Signature verification





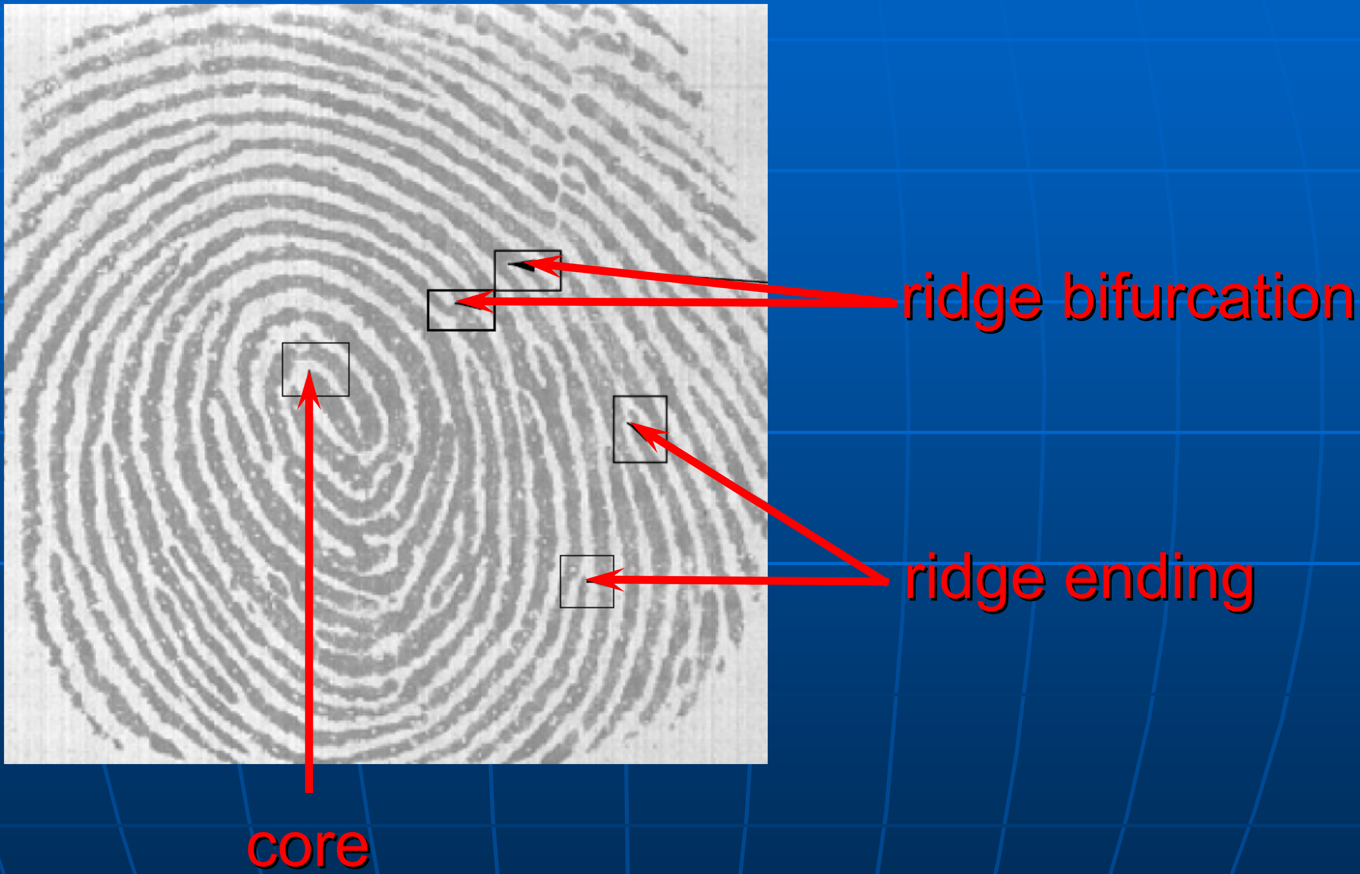
signature before and after skeletonization



detected line-end points and branch-points

L.C. Bastos et a

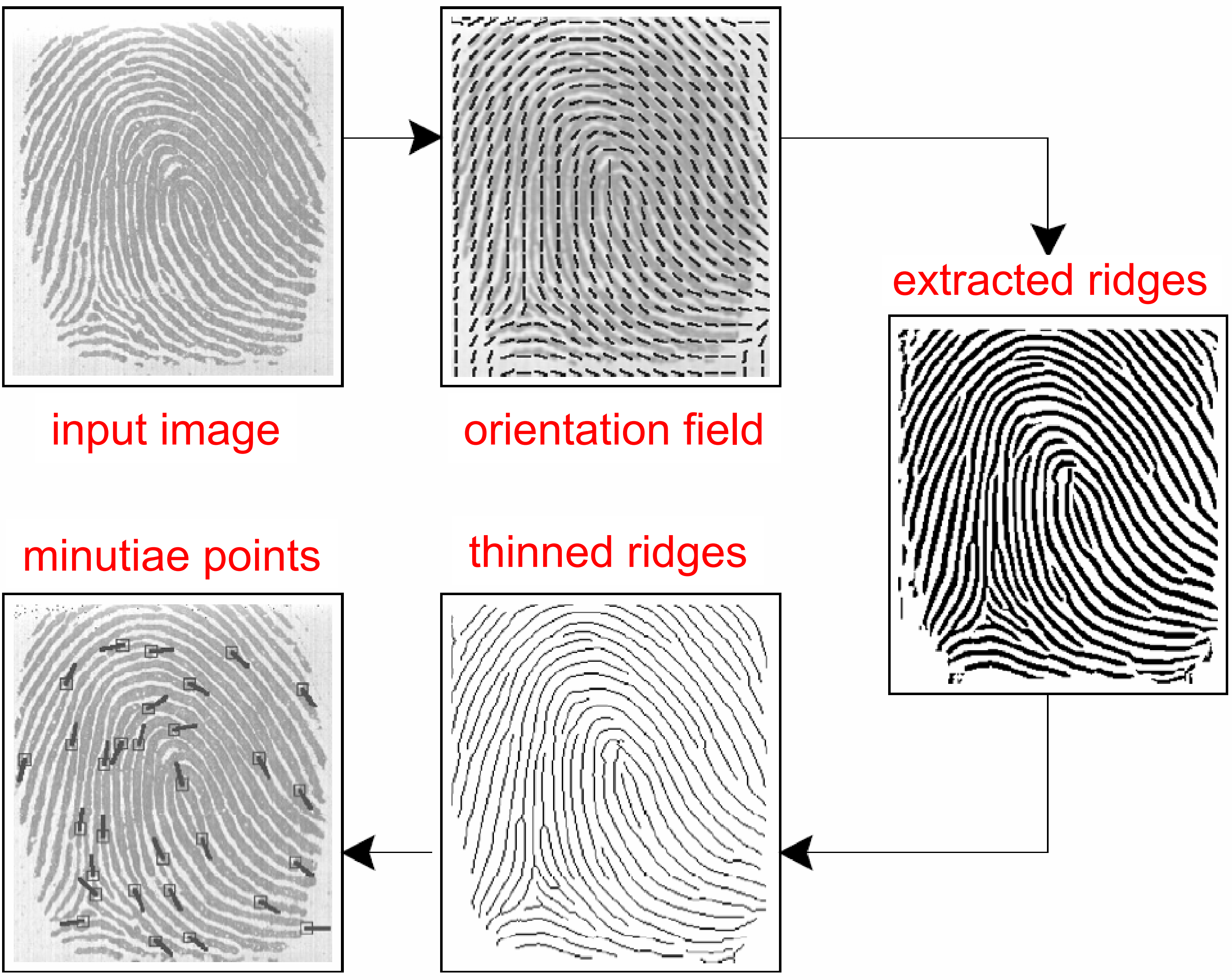
Fingerprint verification



features in fingerprints

A. Ros

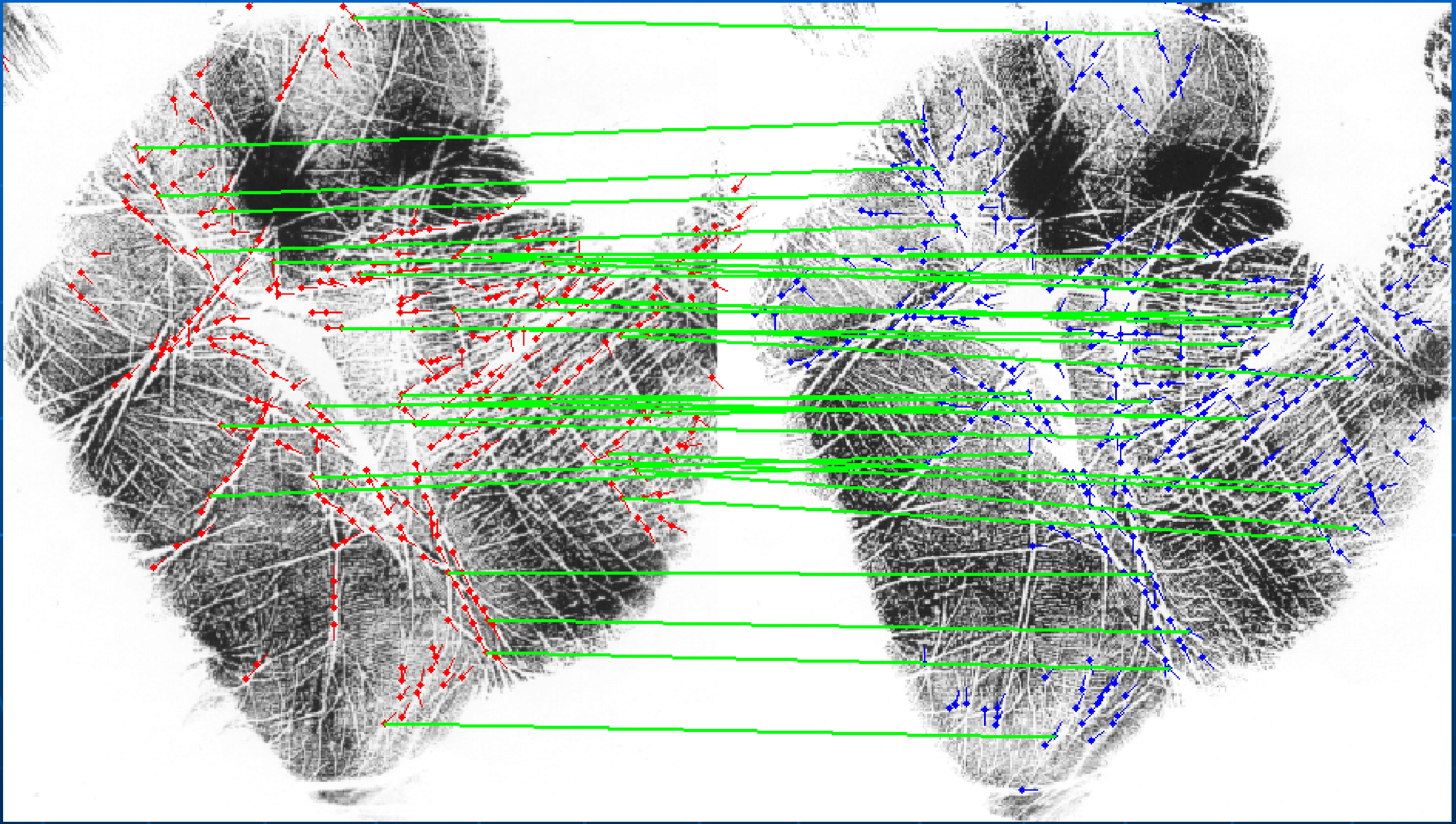
Fingerprint verification



The process

A. Ros

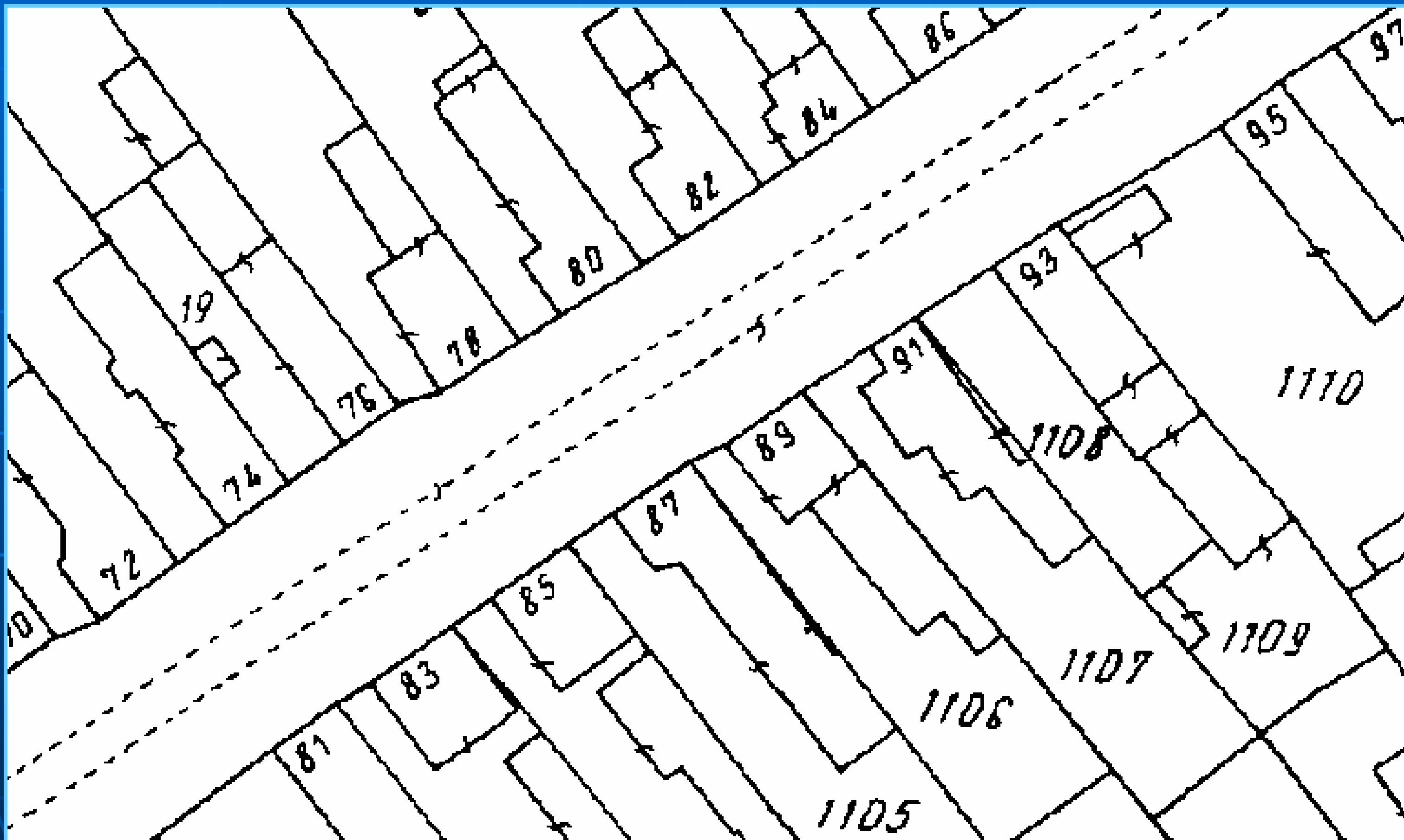
Palmprint verification



matching extracted features

N. Dut

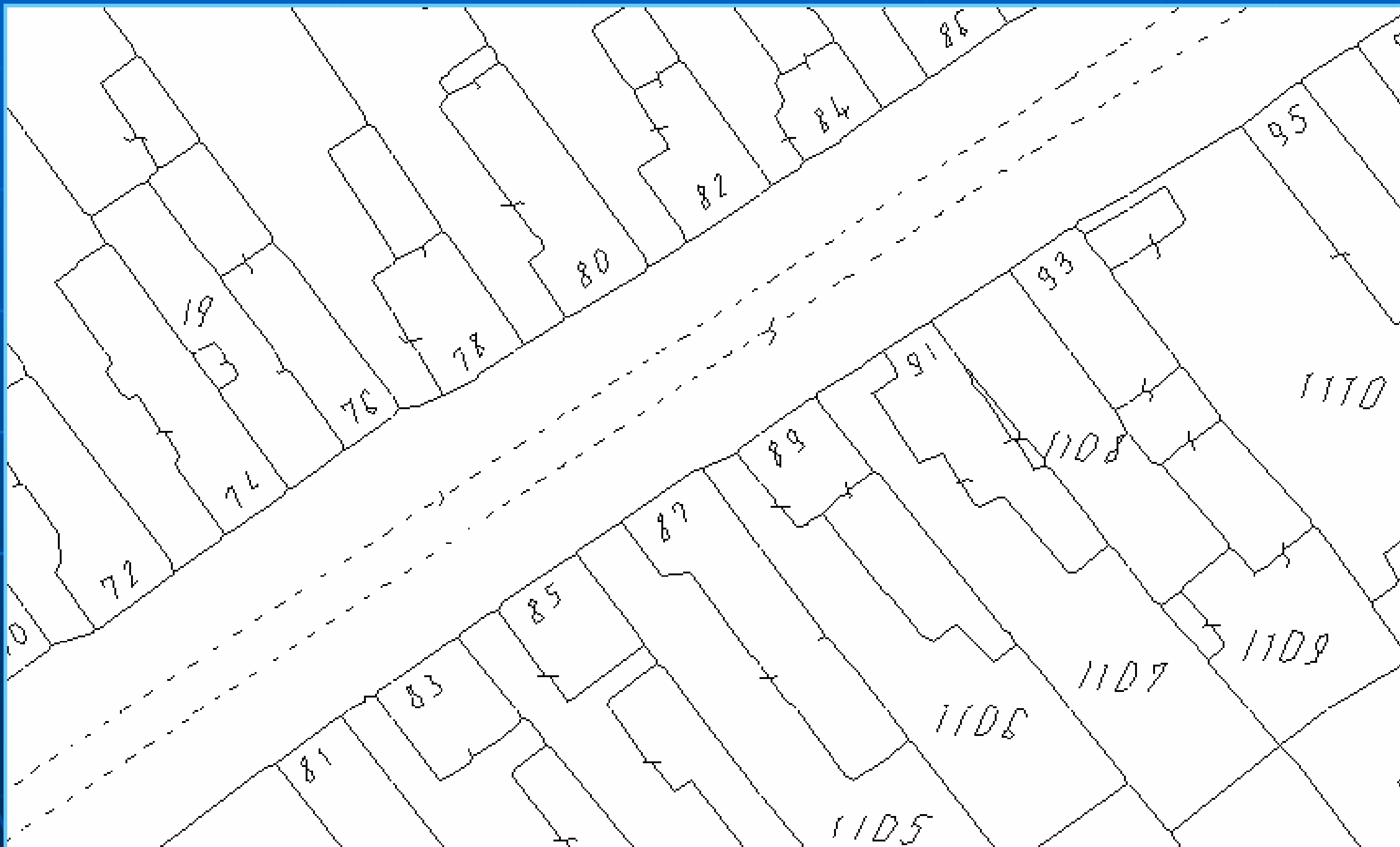
Raster-to-vector conversion



scanned map

Katona E

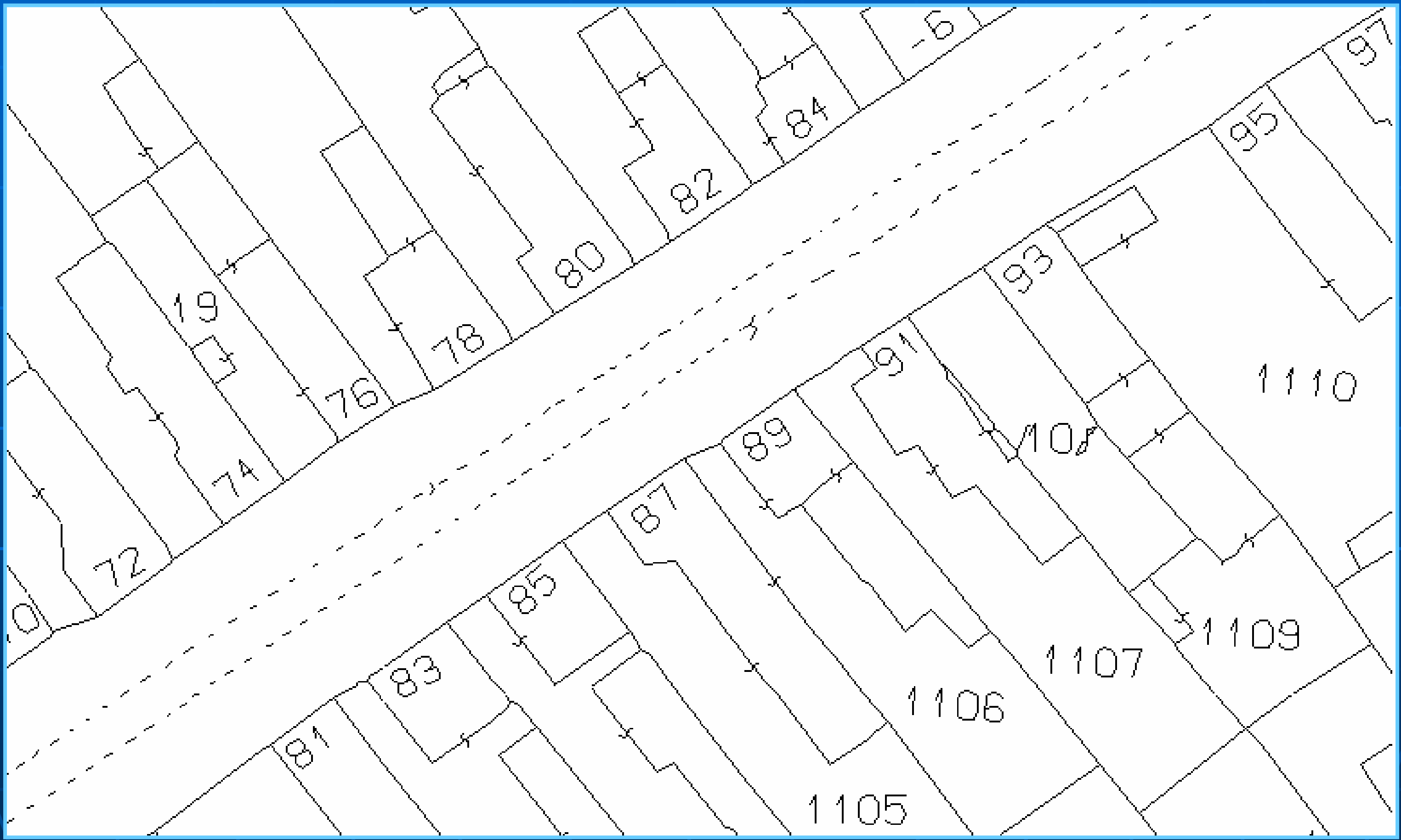
Raster-to-vector conversion



„raw” vector image after skeletonization

Katona E

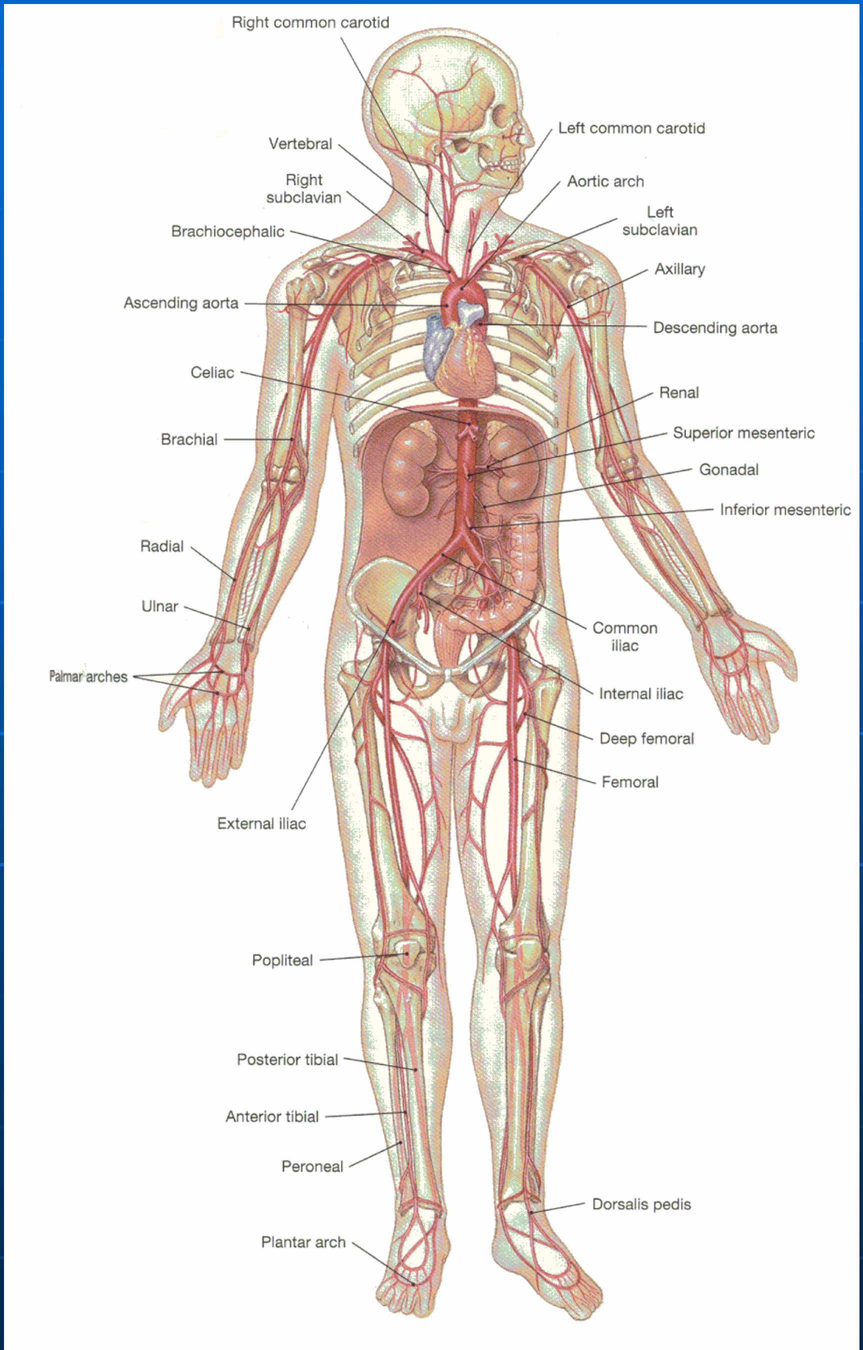
Raster-to-vector conversion



corrected vector image

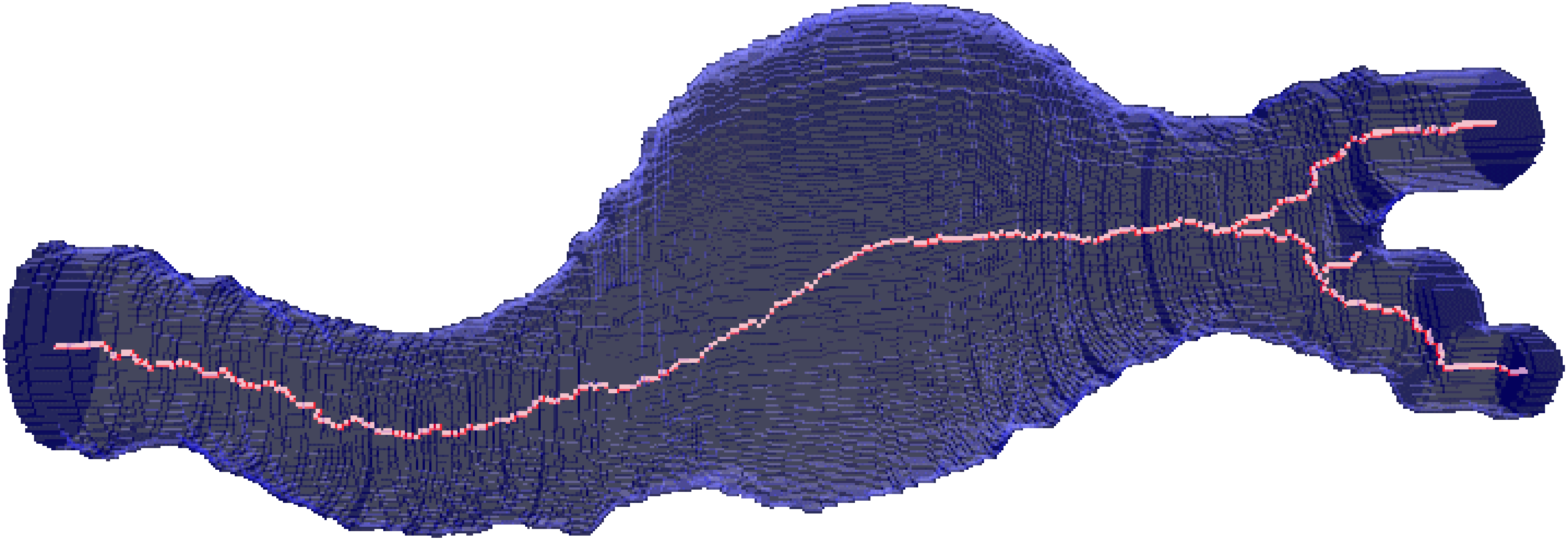
Applications in 3D

There are some frequently used 3D medical scanners (e.g., CT, MR, SPECT, PET), therefore, applications in medical image processing are mentioned.



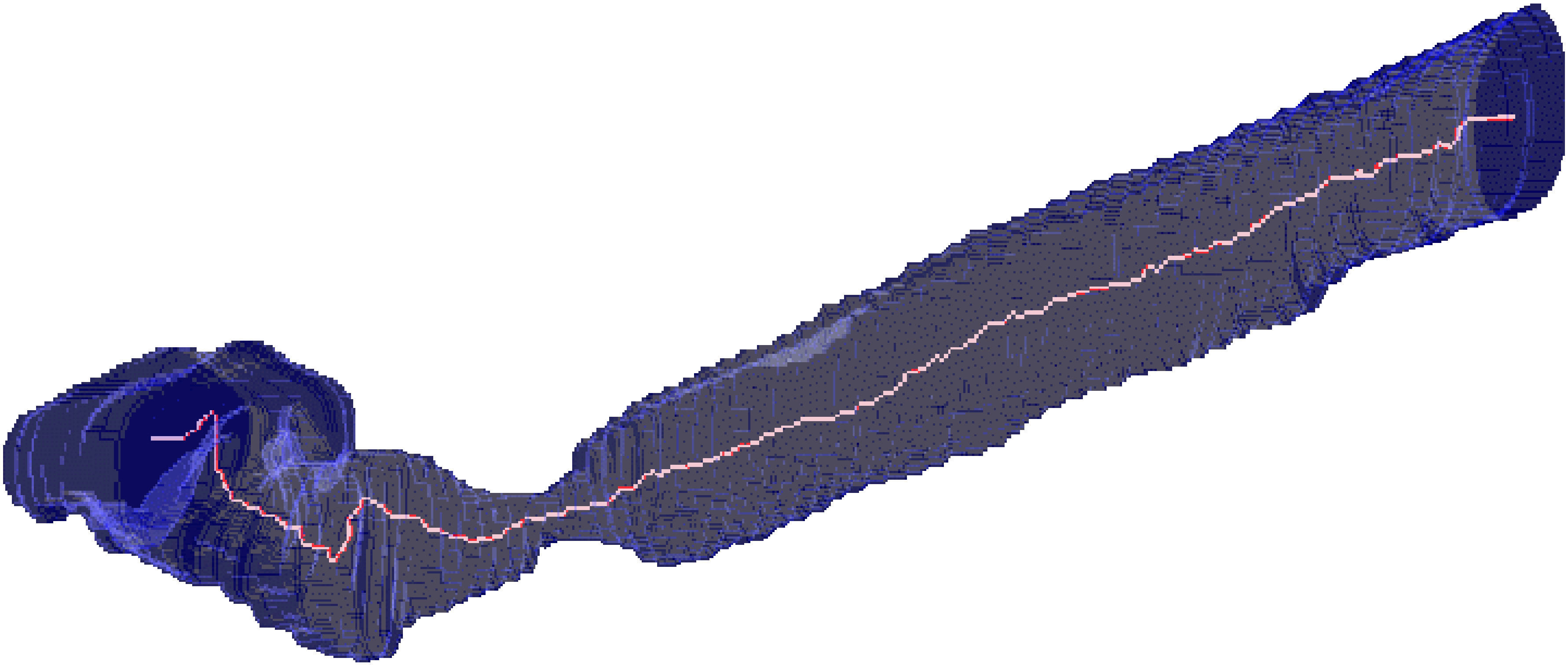
There are a lots of tubular structures (e.g., blood vessels, airways) in the human body, therefore, centerline extraction is fairly important.

Blood vessel
(infra-renal aortic aneurysms)



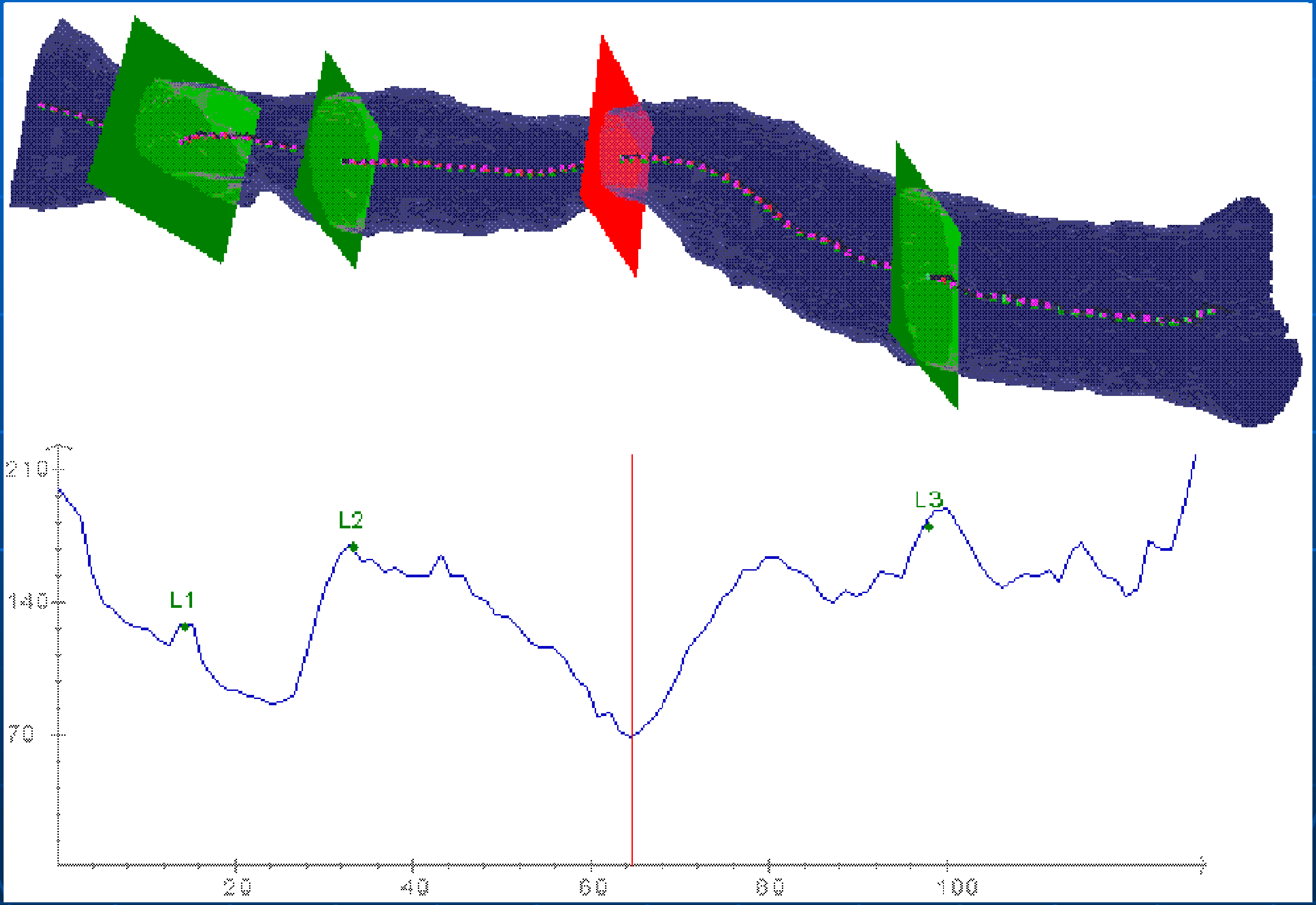
E. Sorantin et al

Airway
(trachealstenosis)



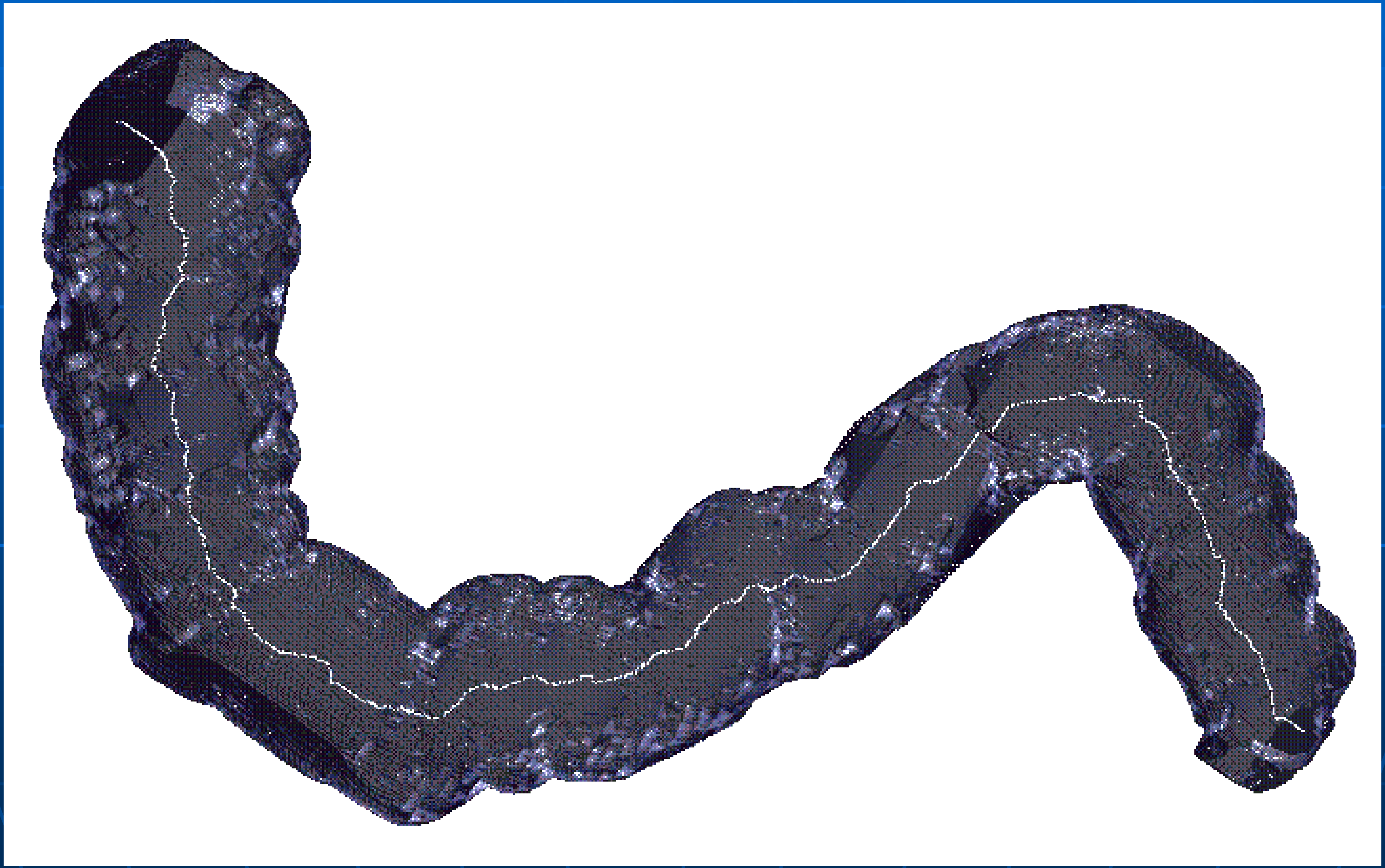
E. Sorantin et al

Airway (trachealstenosis)



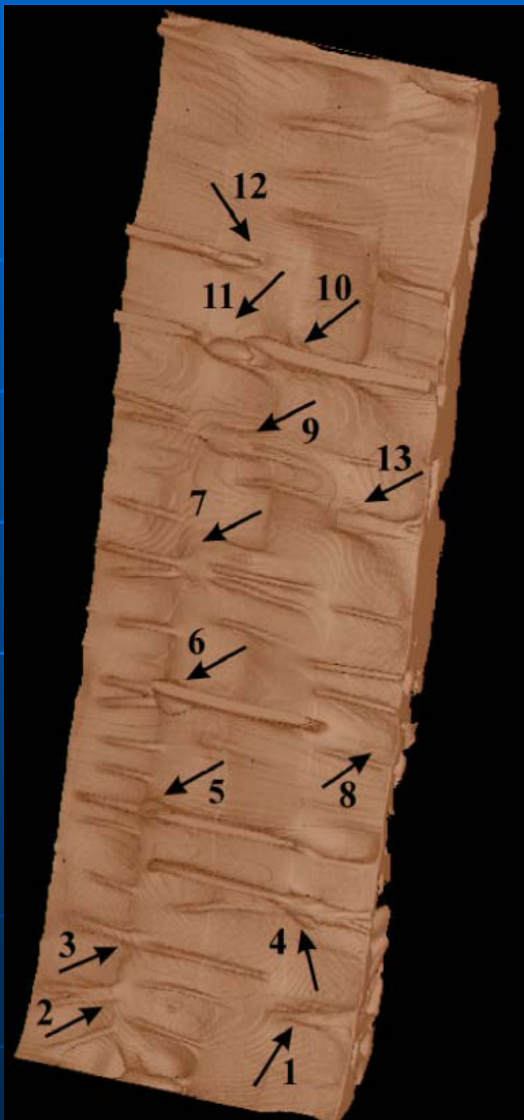
E. Sorantin et a

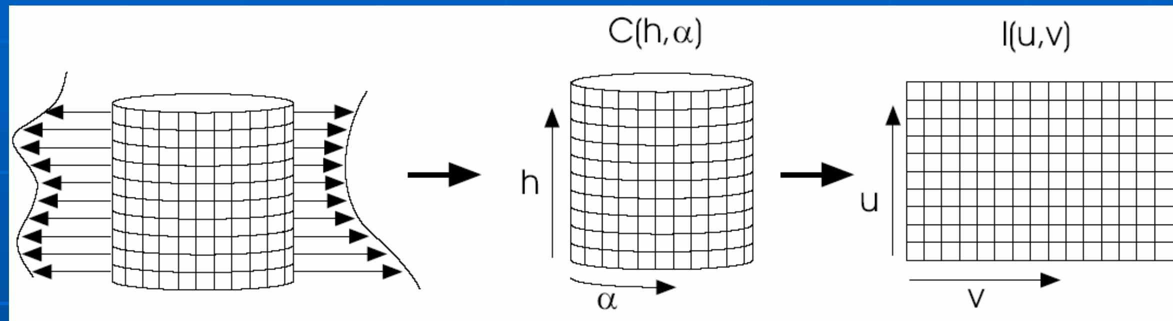
Colon



E. Sorantin et a

Virtual dissection of the colon



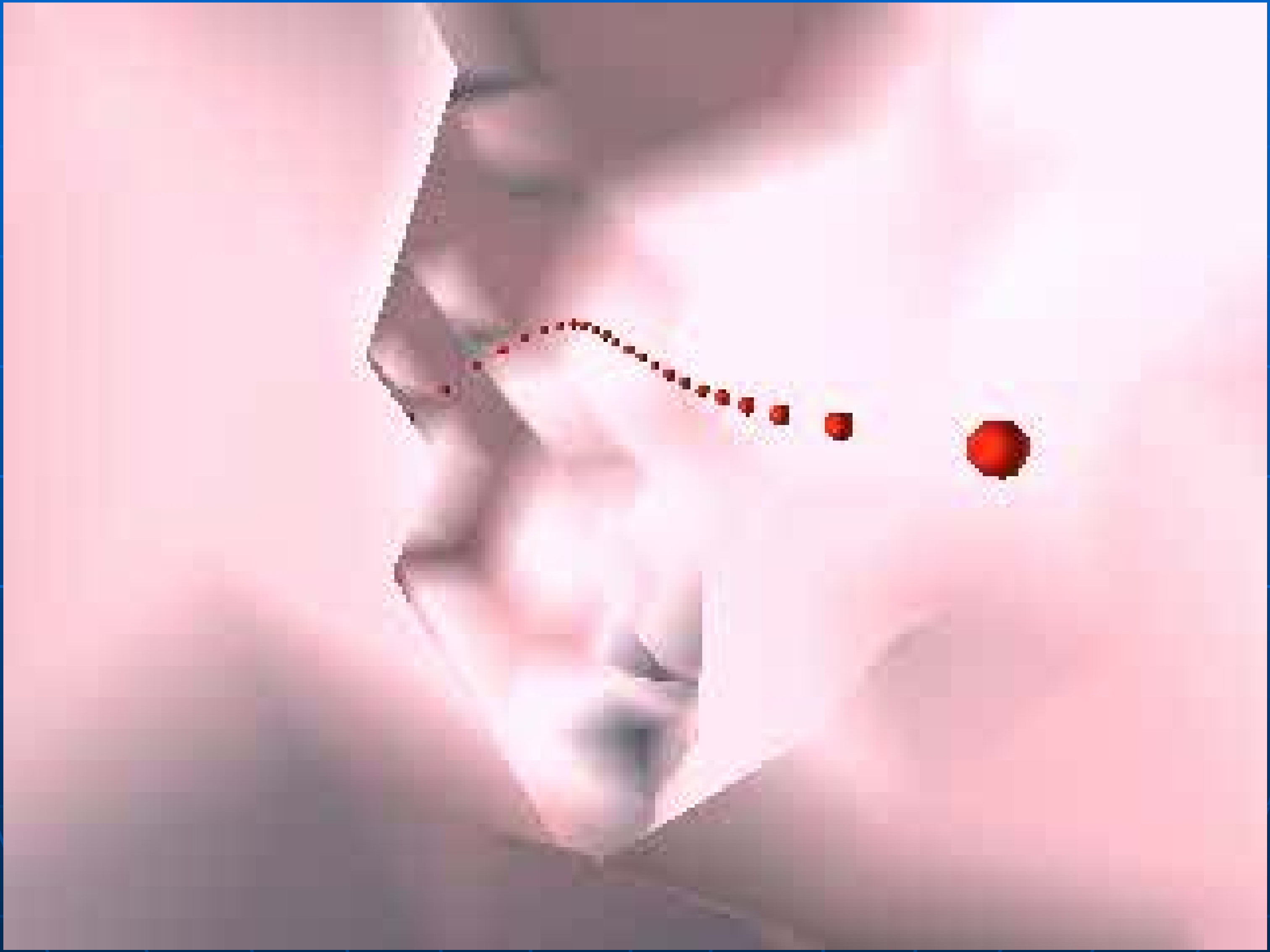


cylindric projection

detected polyps

E. Sorantin et a

Virtual colonoscopy



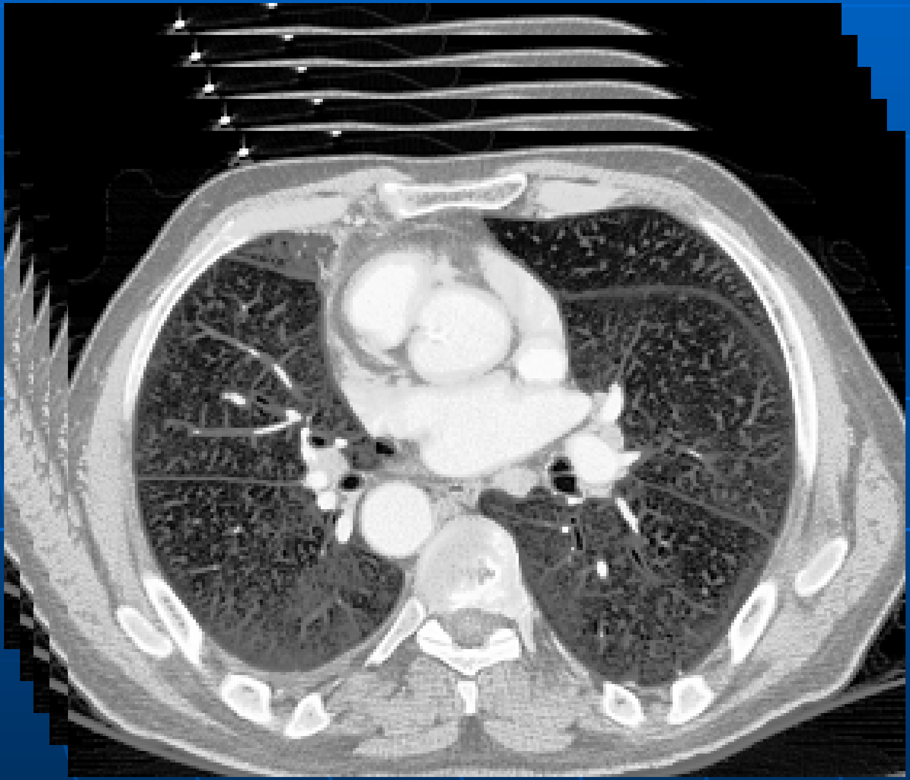
A. Villanova et a

Quantitative analysis of intrathoracic airway trees

Kálmán Palágyi
Juerg Tschirren
Milan Sonka
Eric A. Hoffman

The University of Iowa
College of Engineering Imaging Group

Images



Multi-detector Row Spiral CT

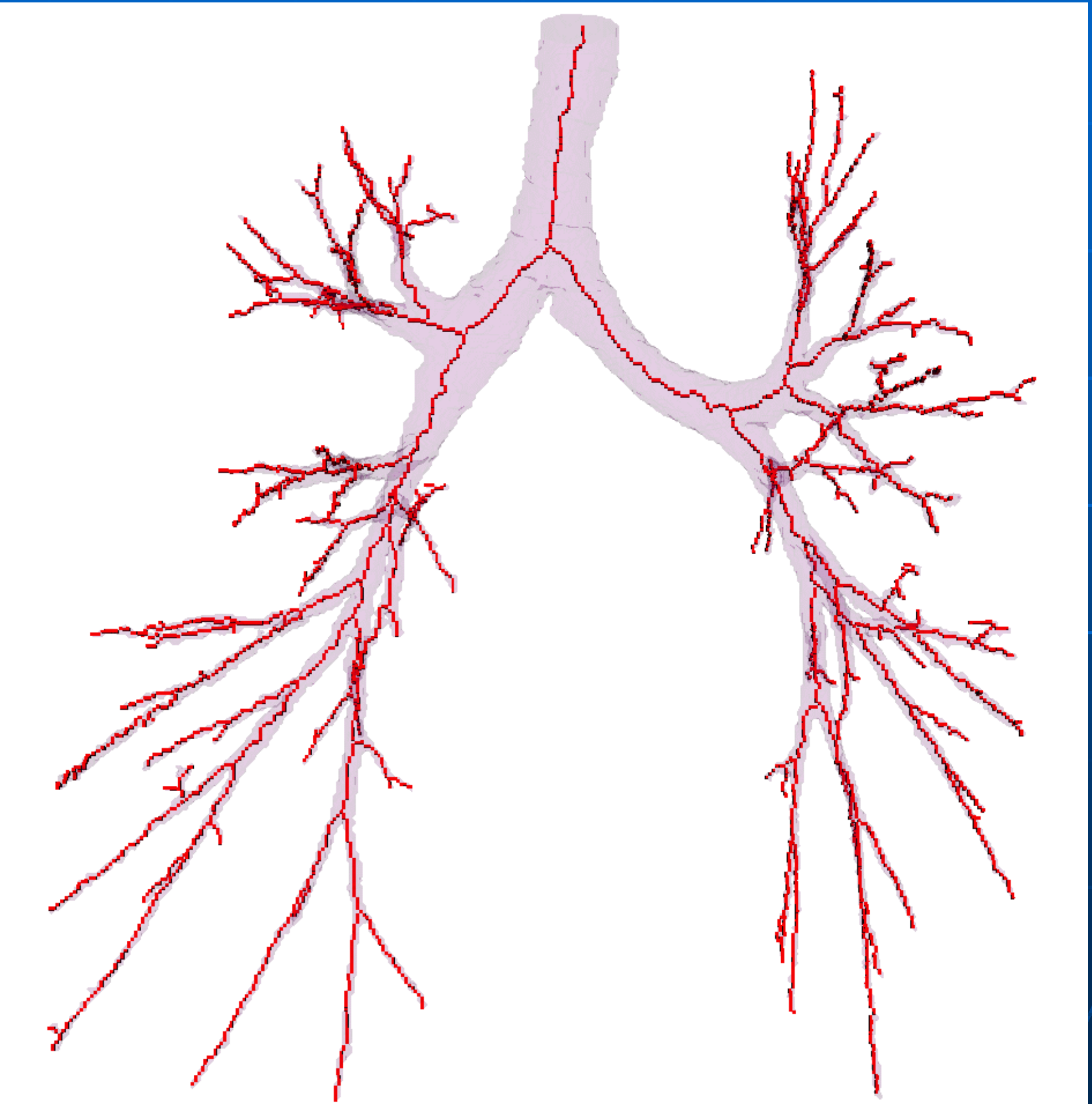
512 x 512 voxels

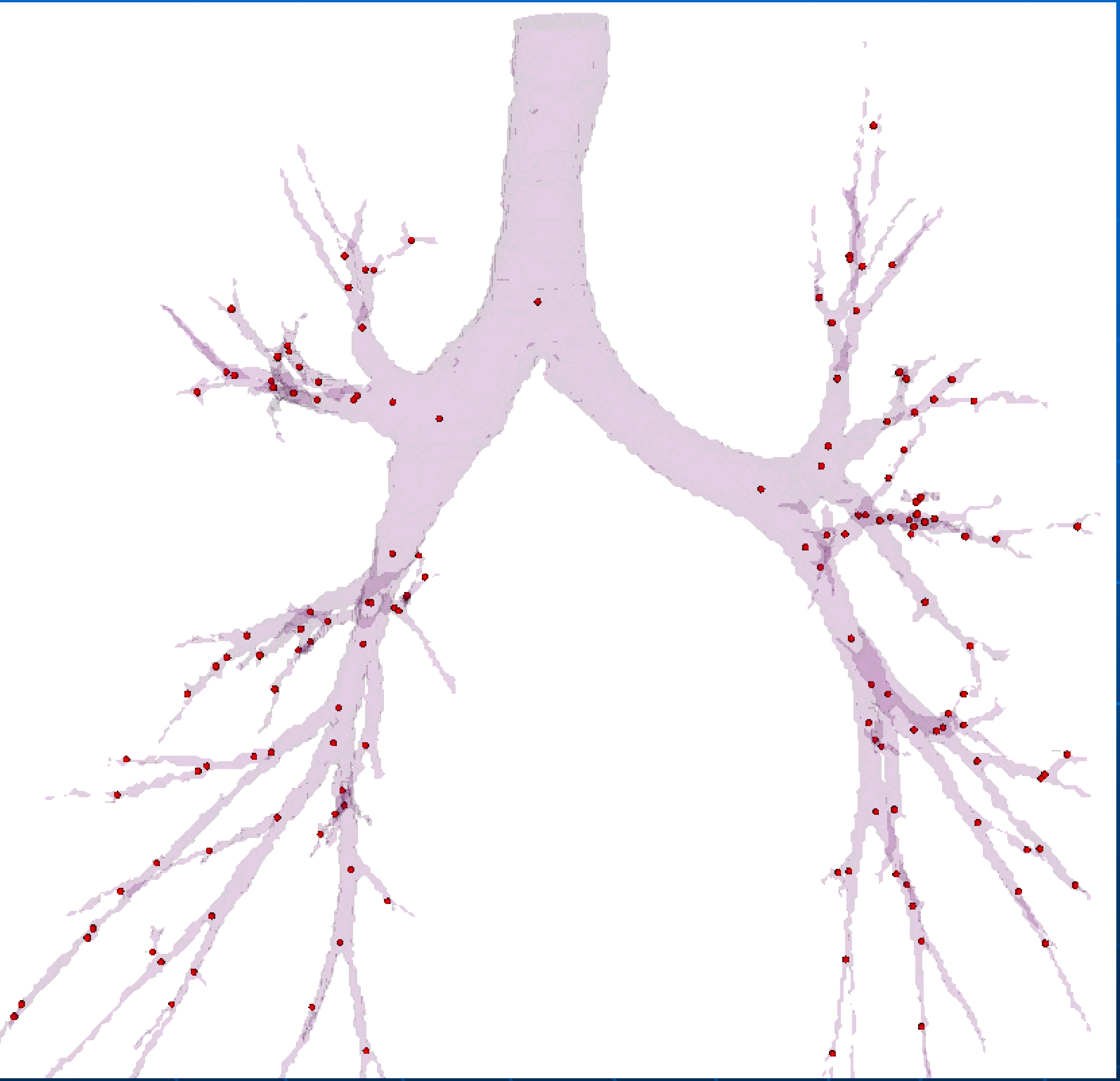
500 – 600 slices

0.65 x 0.65 x 0.6 mm^3 (almost isotropic)

Lung segmentation

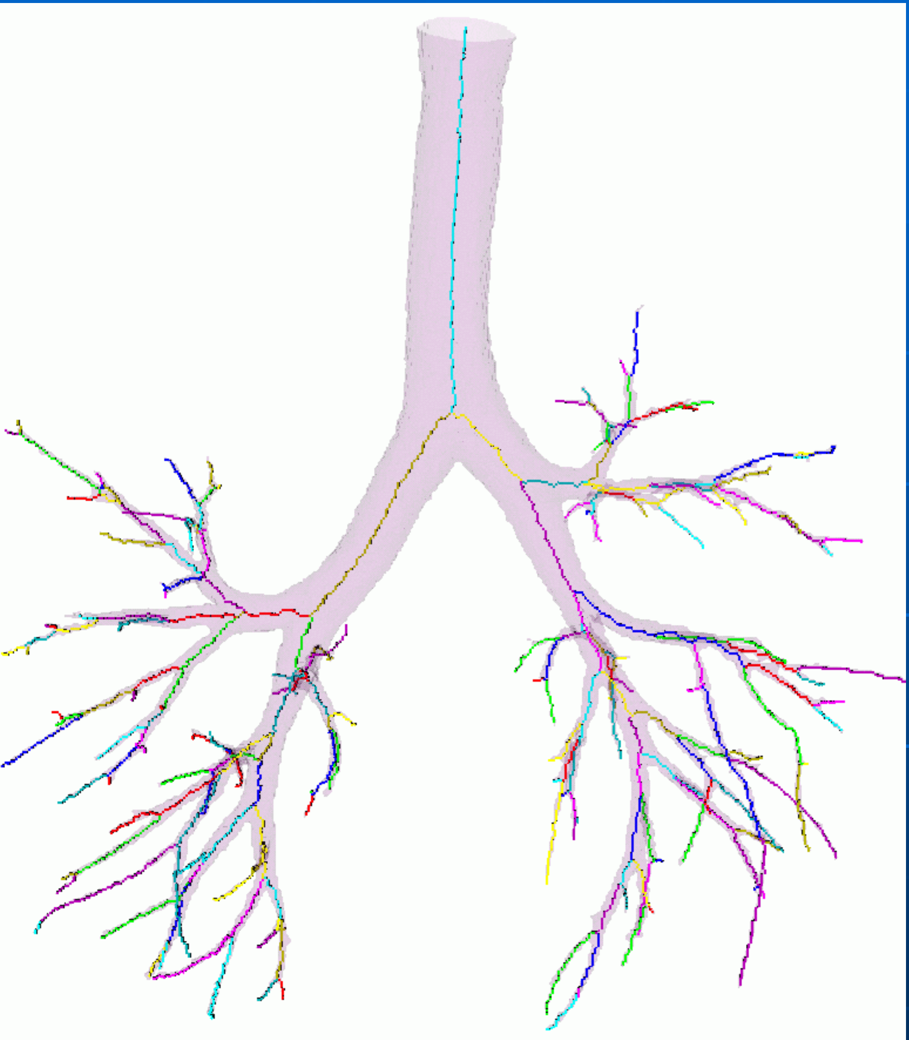
Centerlines

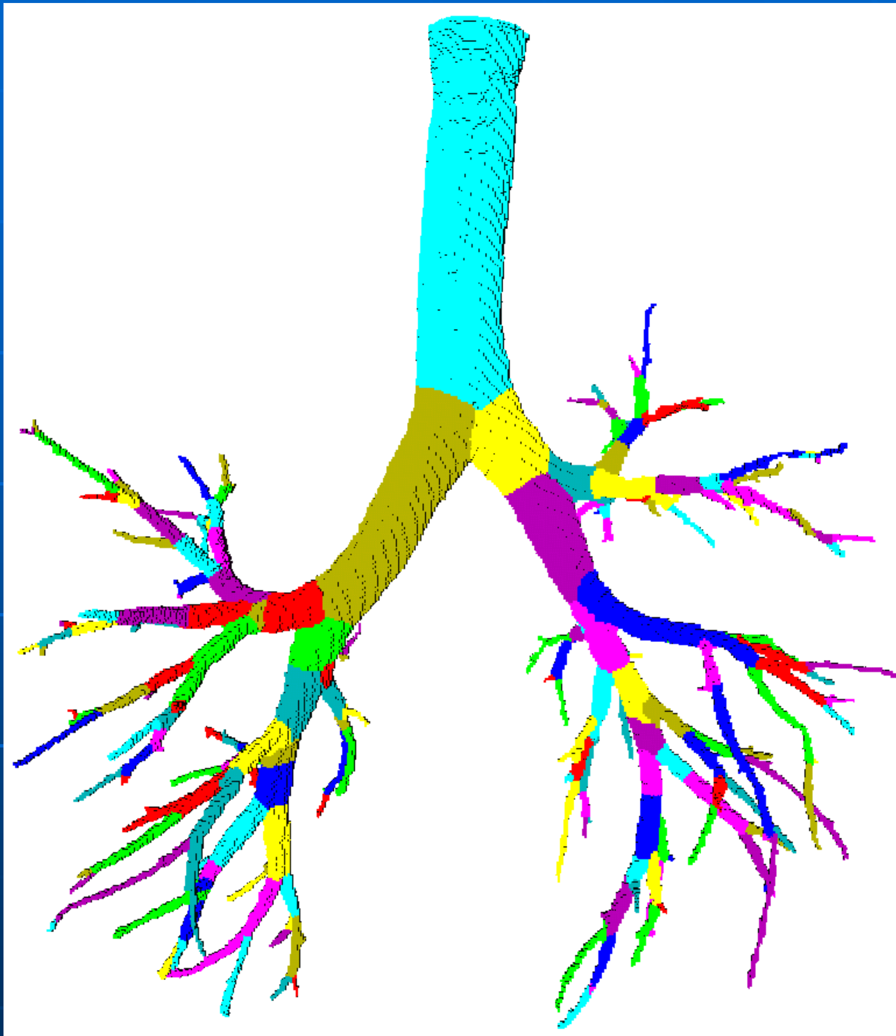




detected branch-points

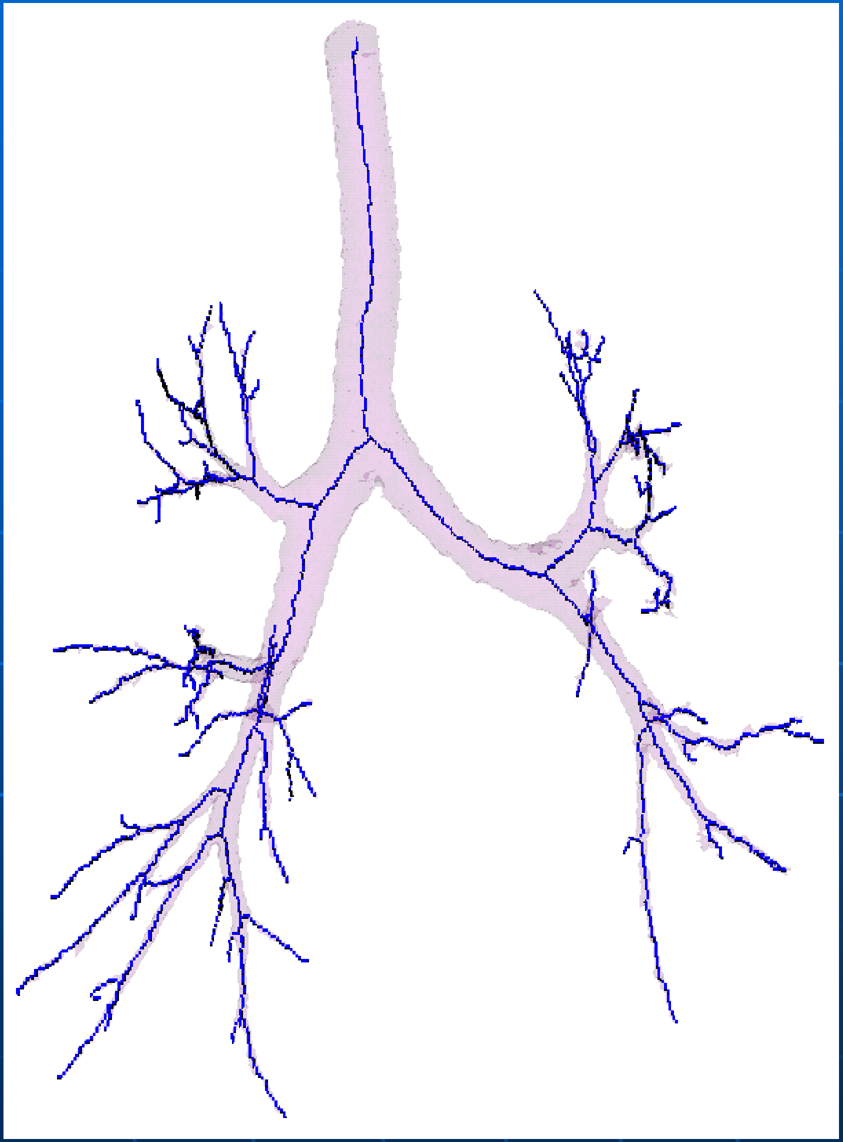
Branch partitioning



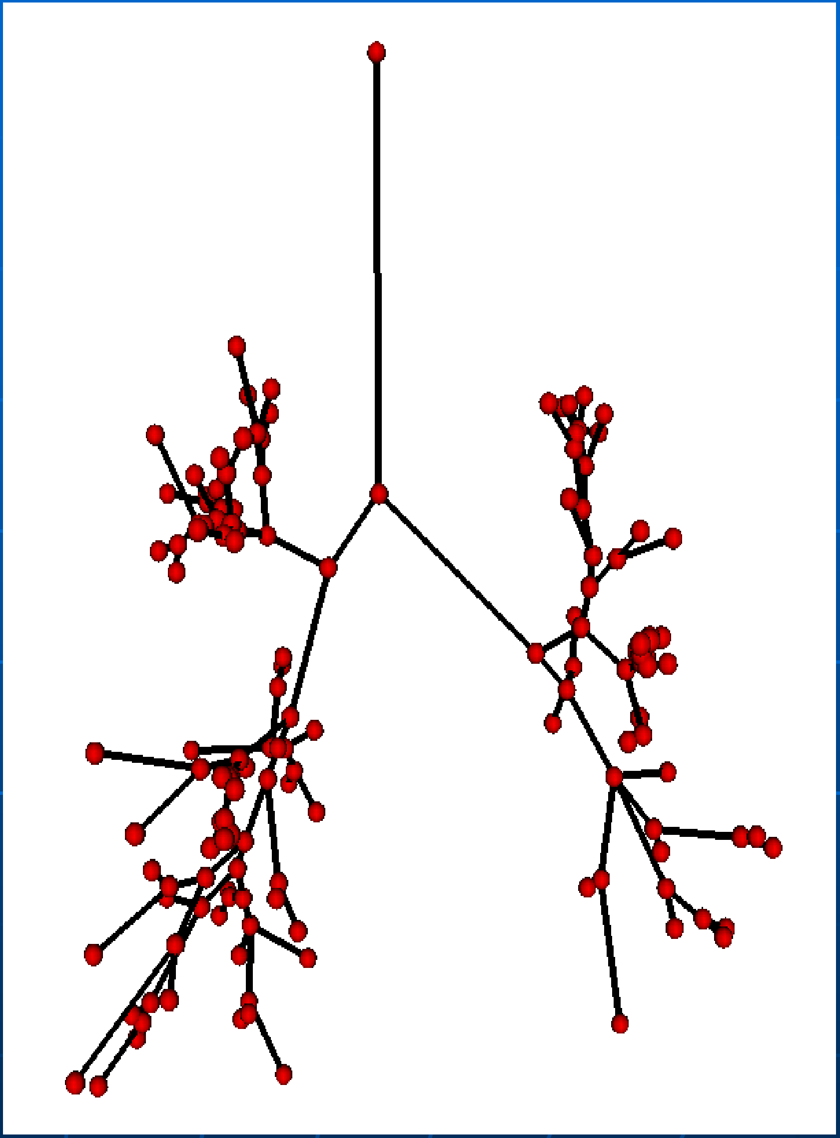


centerline labeling

label propagation



tree with its centerlines



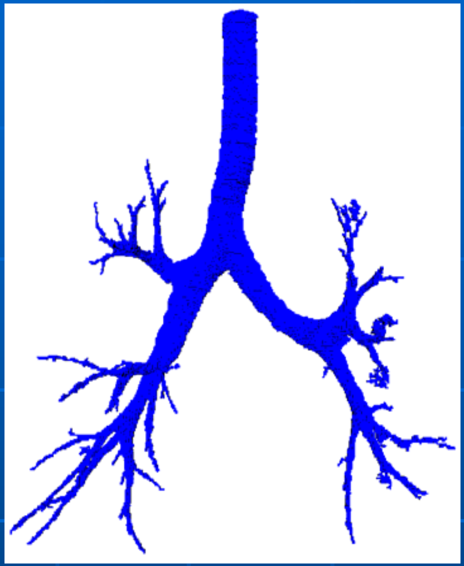
formal tree (in XML)

Quantitative indices for tree branches

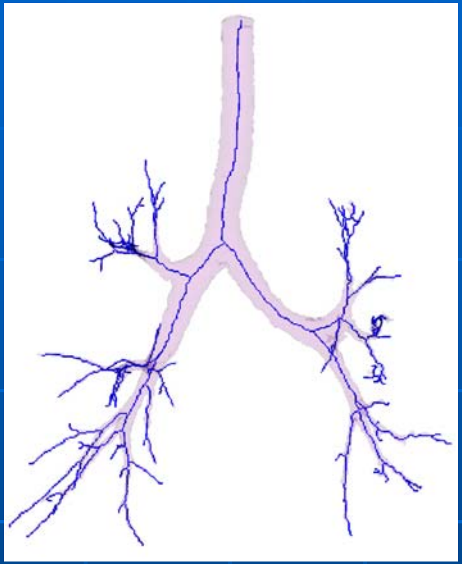
- length (Euclidean distance between the parent and the child branch points)
- volume (volume of all voxels belonging to the branch)
- surface area (surface area of all boundary voxels belonging to the branch)
- average diameter (assuming cylindric segments)

Example of the entire process

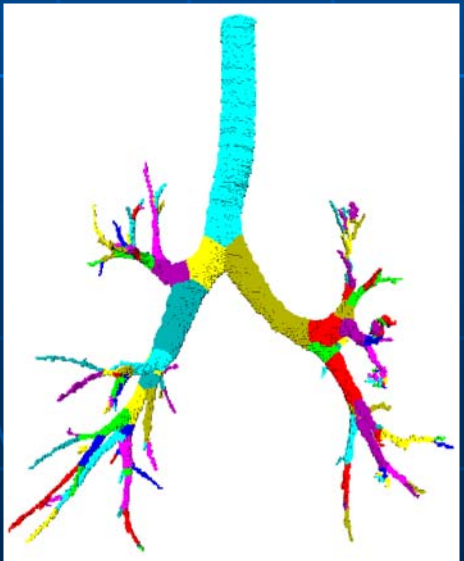
segmented tree



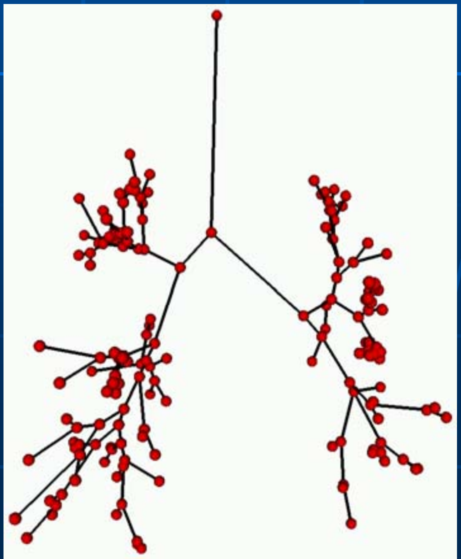
pruned centerlines



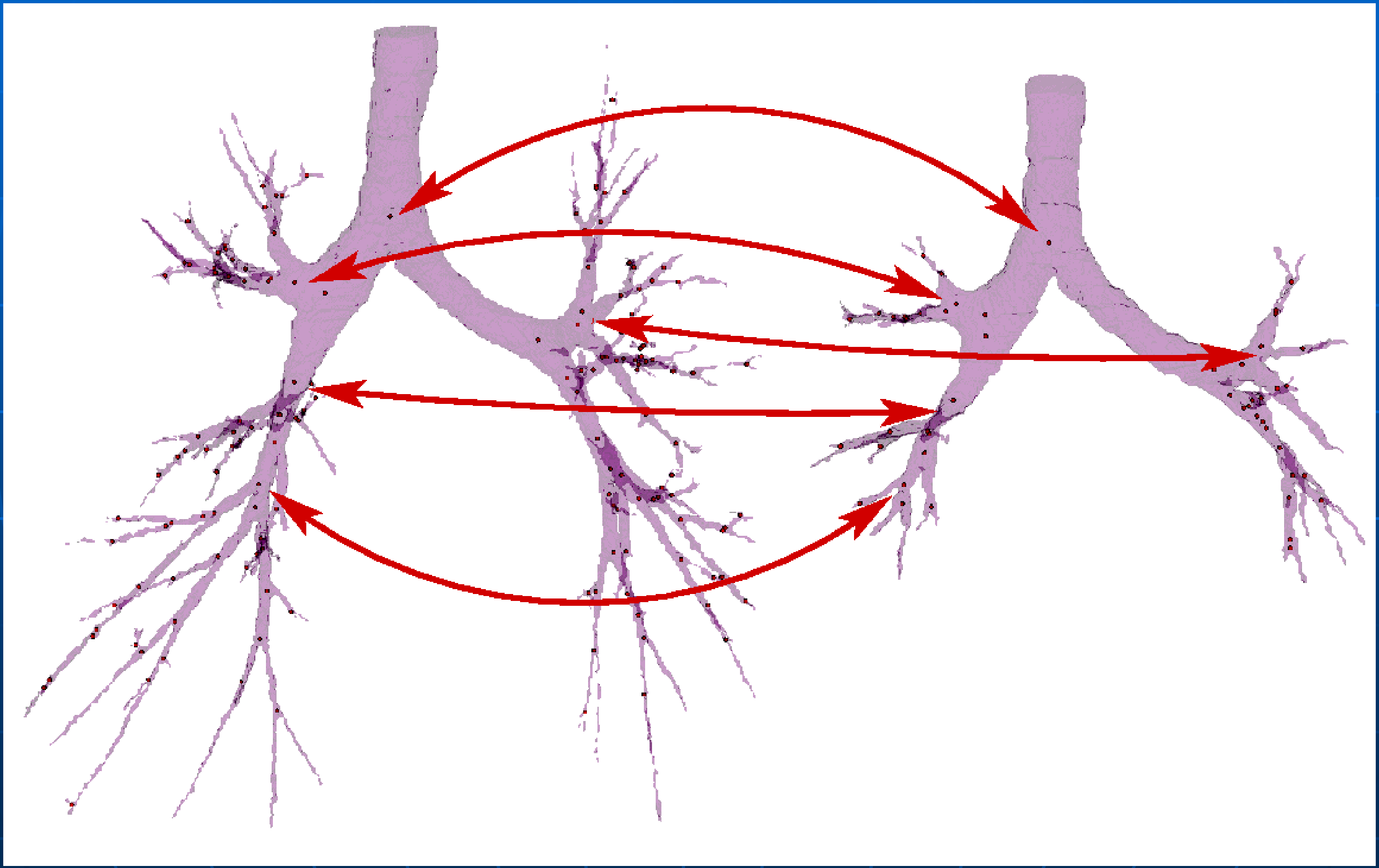
labeled tree



formal tree



Matching



FRC

TLC

Anatomical labeling