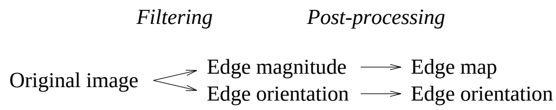
Lecture 6
Template Matching and Feature Detection

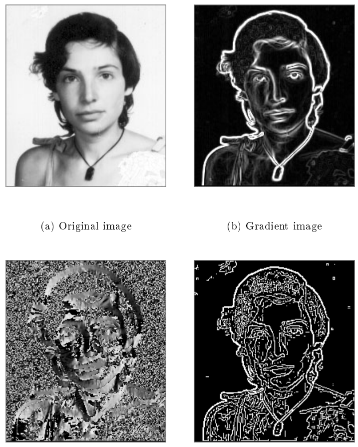
Part 2: Edge detection

- general scheme of edge detection
- gradient operators
- compass operators
- Laplace and zero-crossing operators
- post-processing
  - non-maxima suppression
  - thresholding with hysteresis

General scheme of edge detection :



General scheme of edge detection. Edge magnitude is the strength of edge, e.g., its gradient. Edge map is a binary edge image.



(a) Original image (b) Gradient image

(c) Orientations (d) Edge map

Example of edge detection by 3×3 size Prewitt mask: (a) original image; (b) gradient image; (c) edge orientation image showing intensity-coded orientation angles; (d) final edge map.

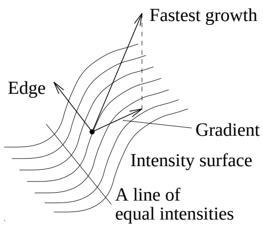
Gradient Operators:

$$Magnitude = \sqrt{f_x^2 + f_y^2}$$

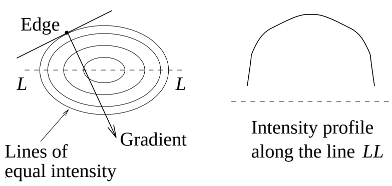
$$Direction = \arctan \frac{f_x}{f_y}$$

Here (f_x, f_y) is the gradient vector formed by the partial derivatives of intensity $f(x, y)$. $Direction$ is the direction, $Magnitude$ the magnitude of the gradient vector. The gradient is orthogonal to the edge.

The magnitude is sometimes approximated by $|f_x| + |f_y|$. This is faster but anisotropic: amplifies the skewed gradients $(\pi/4, 3\pi/4, ...)$ by a factor of $\sqrt{2}$.



Edge detection and gradient for a piece of straight contour.



Gradient, edge orientation and intensity profile for a light blob.

In discrete images, the gradient is approximated by finite differences. Various *convolution masks* are used:

Some Common Gradient Operators

	G_1	G_2
Roberts	$\begin{bmatrix} \boxed{0} & 1 \\ -1 & 0 \end{bmatrix}$	$\begin{bmatrix} \boxed{1} & 0 \\ 0 & -1 \end{bmatrix}$
Prewitt	$\begin{bmatrix} -1 & 0 & 1 \\ -1 & \boxed{0} & 1 \\ -1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & -1 & -1 \\ 0 & \boxed{0} & 0 \\ 1 & 1 & 1 \end{bmatrix}$
Sobel	$\begin{bmatrix} -1 & 0 & 1 \\ -2 & \boxed{0} & 2 \\ -1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & -2 & -1 \\ 0 & \boxed{0} & 0 \\ 1 & 2 & 1 \end{bmatrix}$
Isotropic	$\begin{bmatrix} -1 & 0 & 1 \\ -\sqrt{2} & \boxed{0} & \sqrt{2} \\ -1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & -\sqrt{2} & -1 \\ 0 & \boxed{0} & 0 \\ 1 & \sqrt{2} & 1 \end{bmatrix}$

Boxed element indicates location of origin. $G_{1,2}$ are X, Y components of gradient, except for Roberts where skewed components are used.

- Roberts. Too simplified, non-symmetric, noise sensitive.
- Prewitt. Smoothed, less noise sensitive, symmetric, fast, but anisotropic: skewed directions amplified.
- Sobel. As Prewitt, but the weights reflect proximity to origin.
- Isotropic. As Sobel, but corrected for anisotropy. Less fast.

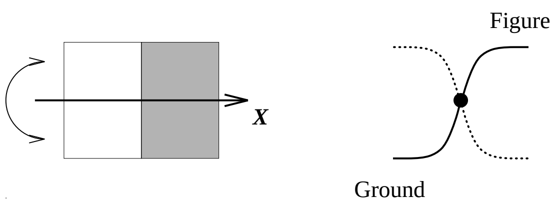
Designing a gradient mask for X direction:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Constraints:

1. Symmetry between left and right:
$$a_{1j} = a_{3j} \quad \text{for } j = 1, 2, 3$$
2. Antisymmetry of edge intensity profile (not necessary):
$$a_{i1} = -a_{i3} \quad \text{and} \quad a_{i2} = 0 \quad \text{for } i = 1, 2, 3$$
3. No response to flat regions:
$$\sum_{i,j} a_{ij} = 0$$

Constraint 2 implies constraint 3. Constraints 1 and 3 are more commonly used than constraint 3.



Designing a 3×3 gradient edge detector using symmetries of the X mask. Left: symmetry between left and right. Right: Antisymmetry of edge intensity profile.

If constraints 1 and 2 are applied, only two free parameters remain:

$$|a_{11}| = |a_{13}| = |a_{31}| = |a_{33}| \equiv p \geq 0$$
$$|a_{21}| = |a_{23}| \equiv q \geq 0$$

The mask becomes

$$\begin{vmatrix} -p & 0 & p \\ -q & 0 & q \\ -p & 0 & p \end{vmatrix}$$

It may also be desirable to *normalise the response* (output) of the operator. For example, to require that for the ideal step edge of unit height the output be 1. The ideal edge has 0's on one side and 1's on the other side, therefore

$$2p + q = 1$$

and the mask has a single free parameter p:

$$\begin{vmatrix} -p & 0 & p \\ 2p-1 & 0 & 1-2p \\ -p & 0 & p \end{vmatrix}$$

For example,

- Prewitt mask: $p = \frac{1}{3}$
- Sobel mask: $p = \frac{1}{4}$
- ...

Compass Operators:

$$Magnitude = \max_k \{|g_k|\}, \quad k = 0, 1, ..., 7.$$

g_k : outputs of 8 masks designed to match 8 basic edge orientations. Each mask is a rotation of the initial mask.

Edge orientation is defined by the rotation of the mask that yields the largest gradient magnitude.

Compass Gradients (North).
Each clockwise circular shift of elements about the center rotates the gradient direction by 45°

$$1)\begin{bmatrix} 1 & 1 & 1 \\ 1 & \boxed{-2} & 1 \\ -1 & -1 & -1 \end{bmatrix} \quad 3)\begin{bmatrix} 1 & 1 & 1 \\ 0 & \boxed{0} & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

$$2)\begin{bmatrix} 5 & 5 & 5 \\ -3 & \boxed{0} & -3 \\ -3 & -3 & -3 \end{bmatrix} \quad 4)\begin{bmatrix} 1 & 2 & 1 \\ 0 & \boxed{0} & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

(Kirsch)

Note: All of the above masks satisfy the symmetry and the zero sum constraints. The first two do *not* satisfy the antisymmetry constraint, that is, do not assume the antisymmetric sigmoid shape of the edge intensity profile.

Eight rotated masks for operator No.3 (Prewitt-like):

1	1	1	↑	1	1	0	↖
0	0	0	(N)	1	0	-1	(NW)
-1	-1	-1		0	-1	-1	
1	0	-1	←	0	-1	-1	↙
1	0	-1	(W)	1	0	-1	(SW)
1	0	-1		1	1	0	
-1	0	1	→	0	1	1	↗
-1	0	1	(E)	-1	0	1	(NE)
-1	0	1		-1	-1	0	
-1	-1	-1	↓	-1	-1	0	↘
0	0	0	(S)	-1	0	1	(SE)
1	1	1		-0	1	1	

Angular resolution: $\frac{\pi}{4}$.

Larger masks ⇒ Higher angular resolution.

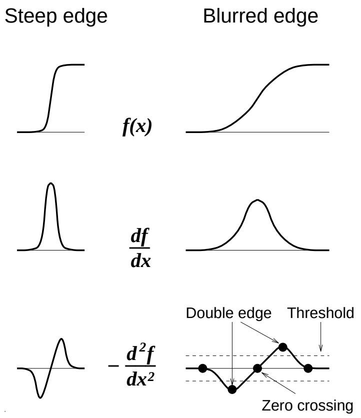
Alternative solution: Define 4 mutually orthogonal masks, *orthogonal gradients*.

Laplace Operator: $g(x, y) = \Delta u(x, y) \equiv \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$

Discrete Laplace Operators

$$1) \begin{bmatrix} 0 & -1 & 0 \\ -1 & \boxed{4} & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad 2) \begin{bmatrix} -1 & -1 & -1 \\ -1 & \boxed{8} & -1 \\ -1 & -1 & -1 \end{bmatrix} \quad 3) \begin{bmatrix} 1 & -2 & 1 \\ -2 & \boxed{4} & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

- Second derivative $\Rightarrow$ Noise-sensitive.
- High-pass filter. May respond to spots and narrow lines as well.
- Yields double edges.
- As edge detector, often used in combination with Gaussian filter. More sophisticated masks are applied.



Use of derivatives for edge detection. Principles of zero-crossing edge detector.

Convolution mask of the *Zero-Crossing Operator*:

$$C \left[\frac{r^2}{\sigma^2} - 1 \right] \exp \left[\frac{-r^2}{2\sigma^2} \right]$$

Here C is a normalisation constant, $r = x^2 + y^2$ is the distance from the mask origin, σ the scale parameter: the smaller the σ the finer the edges obtained.

The zero-crossing mask has the following meaning:

Zero-crossing = Gaussian smoothing + Laplacian edge detection

= Laplacian of (circularly symmetric) Gaussian $G(r) \equiv \exp \left[\frac{-r^2}{2\sigma^2} \right]$

= Second derivative of $G(r)$ with respect to r.

In digital images, a discrete version of the mask is used. The mask becomes larger for larger σ. For example, $\sigma = 4$ needs a mask of about 40 pixels wide.

Edge is indicated in a pixel if a zero-crossing is found in at least one direction across this point. That is, at least one pair of opposite neighboring pixels exists with magnitudes of different signs.

Properties of the zero-crossing operator:

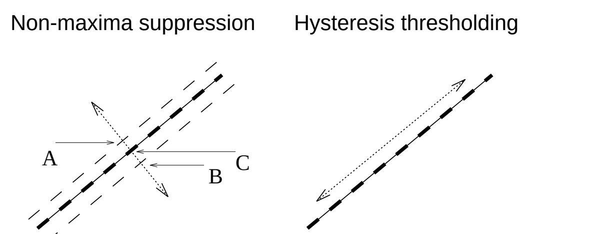
- Smoothed $\Rightarrow$ Less noise-sensitive.
- Closed contours obtained as cross-sections of magnitude surface at zero level. (Good? Spurious loops appear!)
- Controlled operator size $\sigma \Rightarrow$ Edge hierarchy (scale-space).
- Edges may only merge or disappear at rougher scales (larger σ) which facilitates structural analysis.
- Supported by neurophysiological experiments.
- Can be approximated by *difference of two Gaussian masks* $\Rightarrow$ Efficient separable implementation.
- May smooth the shape too much: sharp corners are lost.

Edge detection criteria:

- Detection: miss no important edges; minimise number of spurious responses.
- Localisation: minimise distance between actual and located positions.
- One response: minimise number of multiple responses to single edge.

To fulfill the 'one response' criterion, that is, to remove the 'phantom' edges, two *post-processing operations* are commonly used:

- *Non-maxima suppression*: delete an edge if it has a stronger neighbor across the edge.
- *Thresholding with hysteresis*: remove 'phantom' edges so as to preserve the contour continuity. Do not remove edges whose strength is above a *high threshold*. Remove edges whose strength is below a *low threshold*. Remove an edge whose strength is between the two thresholds if it does not have a strong neighbor along the edge.



Principles of post-processing for edge detection. Non-maxima suppression: remove A and B as they have a stronger neighbor C *across the edge*. Thresholding with hysteresis: threshold with hysteresis *along the edge*.


(a) Prewitt 3x3


(b) Mérő-Vassy 7x7


(c) Zero-crossing 9x9


(d) Deriche 3x3


(e) Deriche 7x7


(f) Deriche 25x25

Examples edge detection: (a) Prewitt 3×3; (b) Mérő-Vassy 7×7; (c) zero-crossing 9×9; (d) Deriche, effective size 3×3; (e) Deriche 7×7; (f) Deriche 25×25.