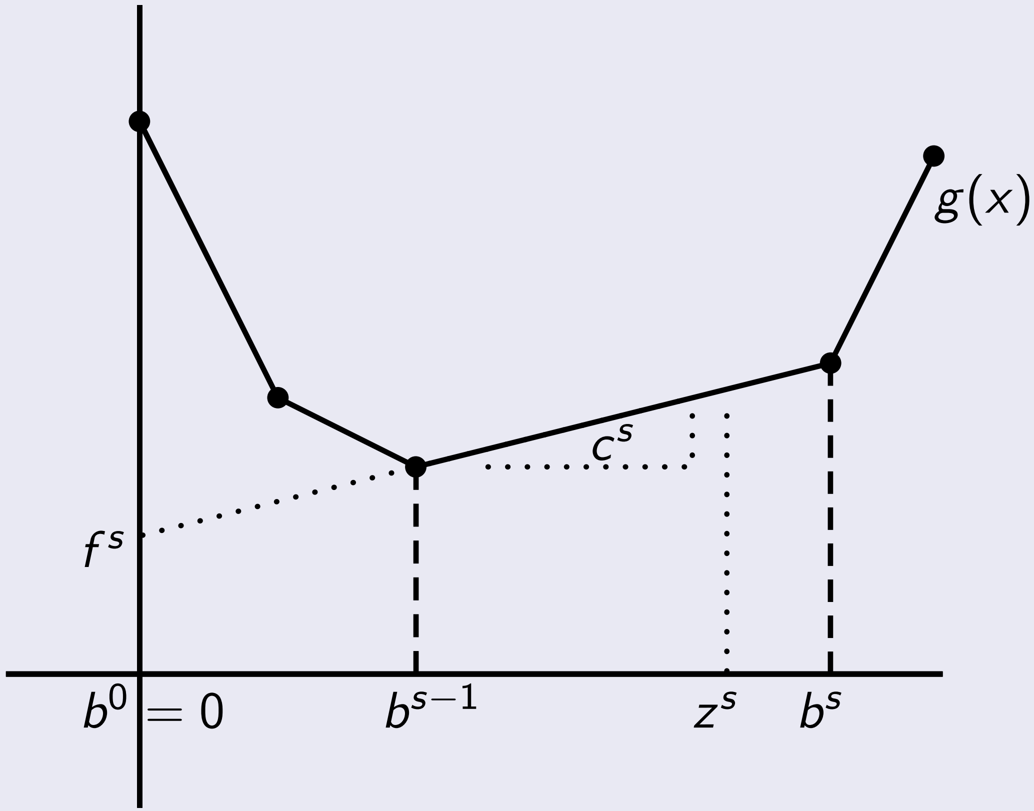
MILP: modeling tricks

Modeling a piecewise linear function

Consider the piecewise linear function $g(x)$ of the picture. Assume that there are $p+1$ breaking points, $b^0, \ldots, b^p$. The slope of the s-th segment $[b^{s-1}, b^s]$ will be denoted by c^s, and the point where the line containing that segment cuts the $0Y$-axis by f^s. Then, the value of $g(x)$ at a point z^s on that segment is given by $g(z^s) = f^s + c^s z^s$.



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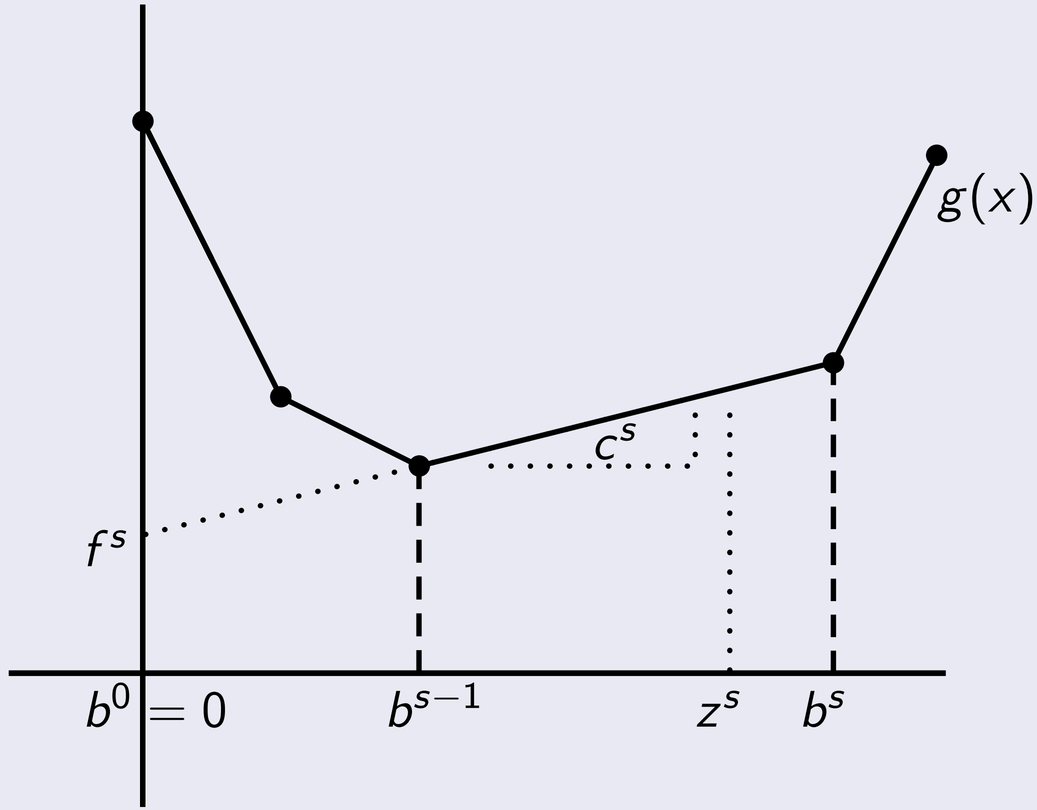
Modeling a piecewise linear function 1

Let us denote

$$z^s = \begin{cases} x & \text{if } x \in [b^{s-1}, b^s] \\ 0 & \text{otherwise} \end{cases} \text{ and } \delta_s = \begin{cases} 1 & \text{if } z^s > 0 \\ 0 & \text{otherwise} \end{cases}, s = 1\ldots,p$$

Then function $g(x)$ can be rewritten as follows:

$$g(x) = \sum_{s=1}^{p}(c^s z^s + f^s\delta_s)$$
$$x = \sum_{s=1}^{p} z^s$$
$$b^{s-1}\delta_s \leq z^s \leq b^s\delta_s$$
$$\sum_{s=1}^{p} \delta_s = 1$$
$$\delta_s \in \{0,1\}, s = 1\ldots,p$$



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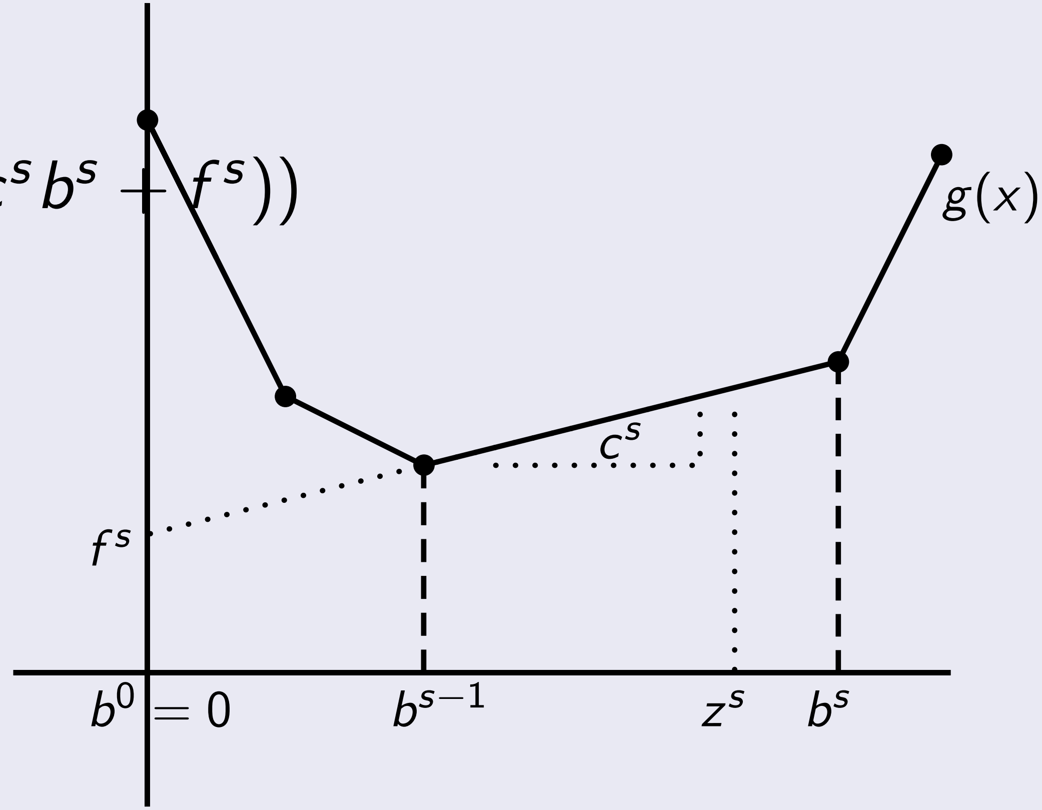
Modeling a piecewise linear function 2

Alternatively, since each point $z^s \in [b^{s-1}, b^s]$ may be written as a convex combination of its end points, $(b^{s-1}, c^s b^{s-1} + f^s)$ and $(b^s, c^s b^s + f^s)$,

$$(z^s, g(z^s)) = \lambda_s(b^{s-1}, c^s b^{s-1} + f^s) + \mu_s(b^s, c^s b^s + f^s), \;\; \lambda_s + \mu_s = 1$$

we can also rewrite the function $g(x)$ as follows:

$g(x) = \sum_{s=1}^{p}(\lambda_s(c^s b^{s-1} + f^s) + \mu_s(c^s b^s + f^s))$
$x = \sum_{s=1}^{p}(\lambda_s b^{s-1} + \mu_s b^s)$
$\lambda_s + \mu_s = \delta_s$
$\sum_{s=1}^{p} \delta_s = 1$
$\delta_s \in \{0, 1\}$
$\lambda_s, \mu_s \geq 0, i = 1, \ldots, p$



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A function must take a value out of N possible values

$$f(x) = b_1 \vee b_2 \vee \ldots \vee b_N$$

can be modeled as

$$f(x) = \sum_{i=1}^{N} b_i \delta_i$$

$$\sum_{i=1}^{N} \delta_i = 1$$

$$\delta_i \in \{0, 1\}, i = 1, \ldots, N$$

MILP: modeling tricks

Transforming integer variables into binary variables

Assume that

$$0 \leq x \leq u, z \in \mathbb{Z}.$$

If $2^N \leq u \leq 2^{N+1}$ then we can represent x using binary variables as follows:

$$x = \sum_{i=0}^{N} 2^i \delta_i, \quad \delta_i \in \{0, 1\}, i = 1 \ldots, N$$