MILP: Why do we need integer variables?

- Sometimes, the variables represent things that, because of its nature, can only take integer values: number of books to buy, number of facilities to be located, number of people to be hired.
- Logic constraints can be modeled via binary/integer variables.
- Some nonlinear models can be approximated using MILP.

Modeling tricks

Dealing with unrestricted variables

If $x_1, \ldots, x_r \in \mathbb{R}$ are unrestricted variables, and our algorithm only works with nonnegative variables, we can change:

$$x_i = x_i^1 - x_i^2, \; x_i^1, x_i^2 \geq 0.$$

This duplicates the number of variables. We can do better and introduce only one additional variable x^* which just move all the variable to the right:

$$x_i = x_i^* - x^*, \; x_i^*, x^* \geq 0.$$

Example

$$\begin{aligned} x_1 + x_2 &\leq 1 \\ 2x_1 - x_2 &\geq 3 \\ x_1, x_2 &\in \mathbb{R} \end{aligned} \quad \text{is equivalent to} \quad \begin{aligned} x_1^* - x^* + x_2^* - x^* &\leq 1 \\ 2(x_1^* - x^*) - (x_2^* - x^*) &\geq 3 \\ x_1^*, x_2^*, x^* &\geq 0 \end{aligned}$$

Modeling tricks

Converting linear equalities into linear inequalities

Using slack and surplus variables we can transform inequalities into equalities. But we can also do the opposite.

$$a_i^t x = b_i, i = 1 \ldots, m$$

can be transformed into

$$a_i^t x \leq b_i, i = 1 \ldots, m$$
$$\left(\sum_{i=1}^m a_i^t\right) x \geq \sum_{i=1}^m b_i$$

Example

$$\begin{aligned} x_1 + x_2 &= 1 \\ 2x_1 - x_2 &= 3 \end{aligned} \quad \text{is equivalent to} \quad \begin{aligned} x_1 + x_2 &\leq 1 \\ 2x_1 - x_2 &\leq 3 \\ 3x_1 &\geq 4 \end{aligned}$$

Modeling tricks

Converting nonlinear objective functions into linear

$$\begin{array}{ll} \min & f(x) \\ s.t. & x \in X \end{array} \quad \text{is equivalent to} \quad \begin{array}{ll} \min & t \\ s.t. & f(x) \leq t \\ & x \in X \end{array}$$

Modeling tricks

Dealing with absolute values

$$\begin{array}{ll} \min & \sum_{i=1}^{m} |f_i(x)| \\ s.t. & x \in X \end{array}$$

is equivalent to

$$\begin{array}{lll} \min & \sum_{i=1}^{m} t_i & \\ s.t. & x \in X & \\ & f_i(x) \leq t_i, & i = 1, \ldots, m \\ & -f_i(x) \leq t_i, & i = 1, \ldots, m \\ & t_i \geq 0, & i = 1, \ldots, m \end{array}$$

Modeling tricks

Dealing with the max function

$$\begin{array}{ll} \min & \max_{i=1}^{m}\{f_i(x)\} \\ s.t. & x \in X \end{array}$$

is equivalent to

$$\begin{array}{ll} \min & t \\ s.t. & x \in X \\ & f_i(x) \leq t, i = 1, \ldots, m \end{array}$$