On set distances and their application to image registration

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What I am going to talk about

- J. Lindblad, V. Ćurić, N. Sladoje. On set distances and their application in image registration. 6th International Symposium on Image and Signal Processing and Analysis ISPA 2009. Accepted

Outline

- Set distances - mathematical background
- Introduce new distances
- Evaluations - having on mind rigid body registration
- Experimental results
- Conclusions and future work

Metric distance

Distance function $d : X \times X \to \mathbb{R}$, on arbitrary set X is a metric if satisfies next four conditions:

1. $d(x, y) \geq 0$ (non-negativity),
2. $d(x, y) = 0 \Leftrightarrow x = y$ (identity of indiscernibles),
3. $d(x, y) = d(y, x)$ (symmetry),
4. $d(x, y) \leq d(x, z) + d(z, y)$ (triangle inequality).

However, for several applications in image processing metricity is not a requirement, and different combinations of conditions (1)–(4) can be well utilized.

Distance from point to the set

- Set distances rely on an underlying distance between the elements of the sets.
- Given an underlying distance $d : X \times X \to \mathbb{R}$, let the distance between $p \in X$ and the non-empty set $A \subset X$ be defined as
$$d(p, A) = \min_{q \in A} d(p, q).$$
- A natural choice of distance between points in space is the Euclidean metric, but in general any point distance can be used.

Hausdorff distance

- The best known and first proposed set distance between two subsets A and B of a metric space (X, d) is the Hausdorff distance, d_H. It is given by

$$d_H(A, B) = \max(\sup_{p \in A} d(p, B), \sup_{p \in B} d(p, A)).$$

- The Hausdorff distance is a metric.

Some practical considerations of Hausdorff distance

- The Hausdorff metric is easy to compute.
- Unfortunately the Hausdorff distance is not suitable for applications where one set has likely a point which is very different from all points of the other set. A single point (outlier) in a set can strongly influence on these distances.
- Hausdorff metric is highly sensitive to noise, especially to background noise of binary image.

Metric by symmetric difference

Distance between two sets A, B is based on the symmetric difference of the sets A and B:

$$A \triangle B = (A \setminus B) \cup (B \setminus A).$$

Metric by Symmetric difference is defined as

$$d_{SD}(A, B) = |A \triangle B| \,,$$

where $|A|$ is the cardinality of the set A.

Some practical considerations of Symmetric difference

- Symmetric difference is easy and simple to compute.
- This metric does not include information of spatial position of point and set, and is therefore not very useful for some applications in image analysis.
- Two non-overlapping objects have maximal distance independent of shape or how far away they are.

Chamfer matching distances

The Chamfer matching distance from set A to set B is defined as

$$d_{CH}^{1}(A,B) = \sum_{p \in \partial A} d(p, \partial B).$$

Squared chamfer matching distance is

$$d_{CH}^{2}(A,B) = \sqrt{\sum_{p \in \partial A} d(p, \partial B)^2}.$$

Chamfer matching distance and Squared chamfer matching distance are directed distances, i.e., symmetry is not satisfied.

Some practical considerations of Chamfer matching distances

- From applicability point of view, chamfer matching distances are easy to implement and faster then other observed distances.
- Chamfer matching distance does not make difference between the object and background of the object. In matching process background of object can be found instead of object.

Sum of minimal distances

Sum of minimal distances (Eiter & Mannila, 1997), d_{SMD}, between the non-empty sets $A, B \subset X$ is defined as,

$$d_{SMD}(A, B) = \frac{1}{2}\Big(\sum_{p \in A} d(p, B) + \sum_{p \in B} d(p, A)\Big).$$

d_{SMD} is not a metric since the triangle inequality is not satisfied.

Directed distances

- Let us introduce the following notation: for a given real constant $r \geq 1$, let

$$\underset{\rightarrow}{d}^{r}(A,B) = \left(\sum_{p \in A} d(p,B)^{r}\right)^{\frac{1}{r}}.$$

- With this notation, $d_{SMD} = \frac{1}{2}(\underset{\rightarrow}{d}^{1}(A,B) + \underset{\rightarrow}{d}^{1}(B,A))$ holds.

Generalization of sum of minimal distances

Definition

Given a distance $d : X \times X \to \mathbb{R}$, non-empty sets $A, B \subset X$, and $r \geq 1 \in \mathbb{R}$, the *Generalized sum of minimal distances* is

$$d^r_{SMD}(A,B) = \frac{1}{2}\Big(\underset{\rightarrow}{d}^{\,r}(A,B) + \underset{\rightarrow}{d}^{\,r}(B,A) \Big), \quad r \geq 1.$$

Connections between observed distances and Sum of minimal distances

- If we take the discrete metric, defined on a nonempty set X as $d(x,y) = \begin{cases} 1, & x \neq y \\ 0, & x = y \end{cases}$, $\quad x, y \in X$ then

$$d_{SMD}(A,B) = \frac{1}{2} d_{SD}(A,B).$$

- For Hausdorff distance holds

$$\frac{1}{2} d_H(A,B) \leq \lim_{r\to\infty} d^r_{SMD}(A,B) \leq d_H(A,B).$$

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$$d_{SMD}(\partial A, \partial B) = \frac{1}{2}\left(d^1_{CH}(A,B) + d^1_{CH}(B,A)\right)$$

Complement distance

- Given a set distance d, a universe U and a complement operator $\overline{X} = U \setminus X$, let the *complement set distance* $\overline{d}$ of d be
$$\overline{d}(A, B) = d(\overline{A}, \overline{B}) \,.$$

Evaluations

- For applicability point of view it is desirable that a small amount of noise has only small impact on the distance value.
- Having registration on mind as an application, we perform our evaluation of considered distances with respect to translation and rotation.
- To ensure successful registration it is desirable that the distance increases monotonically with increasing translation and rotation.

Evaluation of noise sensitivity

To evaluate noise sensitivity of the distances we add three types of noise.

- Pepper noise - removal of 5% of the pixel randomly inside of the object.
- Salt and pepper noise - change of value for 1% of pixels in the whole image.
- Boundary noise - random pixels values are assigned to inner or outer boundary pixels.

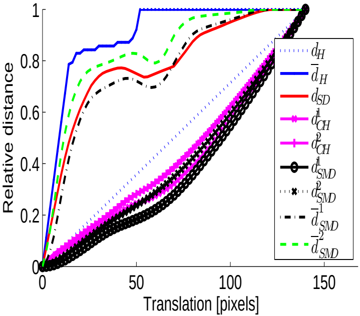
Tests

Test is performed on 40 images.

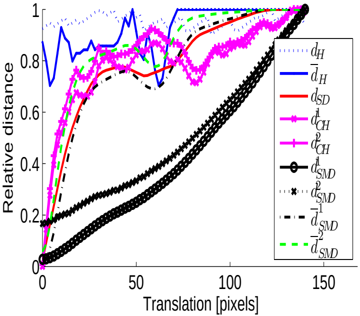
Example of test images.

- Translation is performed in horizonal step of one pixel starting from zero up to twice width of the object.
- Rotation is done in the range of $[0, \frac{\pi}{4}]$.

Results



Relative distance
Translation [pixels]
0
0.2
0.4
0.6
0.8
1
0
50
100
150
d_H
$\bar{d}_H$
d_{SD}
d^1_{CH}
d^2_{CH}
d^1_{SMD}
d^2_{SMD}
$\bar{d}^1_{SMD}$
$\bar{d}^2_{SMD}$



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d_{SD}
d^1_{CH}
d^2_{CH}
d^1_{SMD}
d^2_{SMD}
$\bar{d}^1_{SMD}$
$\bar{d}^2_{SMD}$

Results of translation

Percent of images where distance is non-decreasing w.r.t. translation, for different types of noise.

	No noise	Pepper	Salt and pepper	Boundary
d_H	**100**	**100**	0	**100**
$\overline{d}_H$	88	0	0	63
d_{SD}	53	53	3	55
d^1_{CH}	98	98	0	98
d^2_{CH}	98	**100**	0	**100**
d^1_{SMD}	98	98	**95**	**100**
d^2_{SMD}	**100**	**100**	78	**100**
$\overline{d}^1_{SMD}$	58	58	0	63
$\overline{d}^2_{SMD}$	60	58	3	63

Results of rotation

Percent of images where distance is non-decreasing w.r.t. rotation, for different types of noise.

	No noise	Pepper	Salt and pepper	Boundary
d_H	40	40	0	35
$\overline{d}_H$	43	3	8	60
d_{SD}	63	65	60	68
d^1_{CH}	73	68	0	73
d^2_{CH}	73	70	0	78
d^1_{SMD}	78	75	53	75
d^2_{SMD}	78	**78**	30	78
$\overline{d}^1_{SMD}$	75	70	**68**	75
$\overline{d}^2_{SMD}$	**80**	58	60	**80**

Conclusions and future work

- We introduce family of new distances - Generalized sum of minimal distances.
- Proposed distances can be useful in image registration.
- Future work will include combination of directed and complement distances.
- Really application in image registration.