Skeletonization and its applications

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Syllabus

- Shape
- Shape features
- Skeleton
- Skeletonization
- Applications

Different shapes

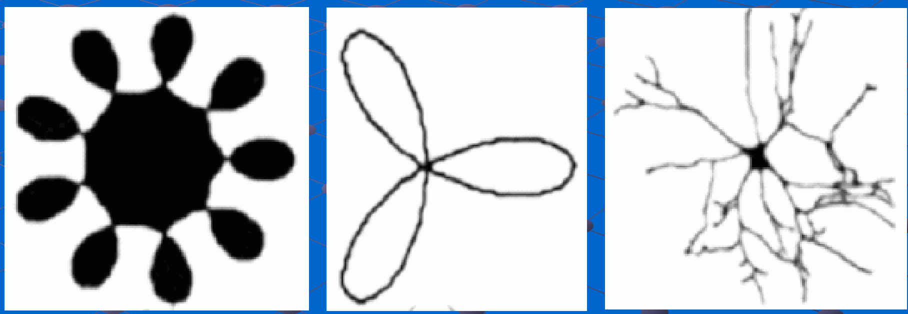
Shape

It is a fundamental concept in computer vision.

It can be regarded as the basis for high-level image processing stages concentrating on scene analysis and interpretation.

Shape

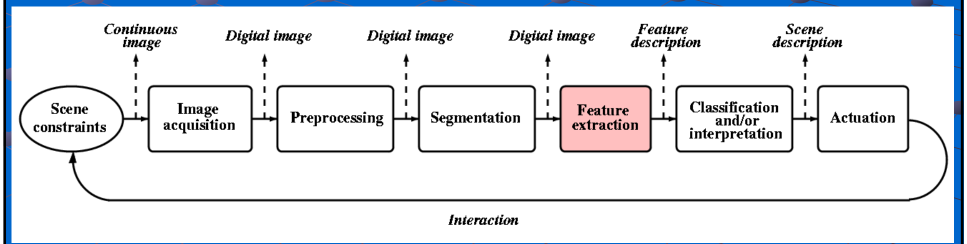
It is formed by any connected set of points.



examples of planar shapes

(L.F. Costa, R. Marcondes, 2001)

The generic model of a modular machine vision system



(G.W. Awcock, R. Thomas, 1996)

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Shape representation

- to apply a transform in order to represent an object in terms of the transform coefficients,
- to describe the boundary that surrounds an object,
- to describe the region that is occupied by an object.

Transform-based shape representation

- Fourier description
- spherical harmonics – based description (3D)
- wavelet-based analysis
- scale-space / multiscale characterization
- …

Contour-based shape representation

- chain-code
- run-length
- polygonal approximation
- syntactic primitives
- spline
- snake / active contour
- multiscale primitives
- …

Region-based shape representation

- polygon
- Voronoi / Delaunay
- quadtree
- morphological decomposition
- convex hull / deficiency
- run-length
- distance transform
- skeleton
- …

Region-based shape representation

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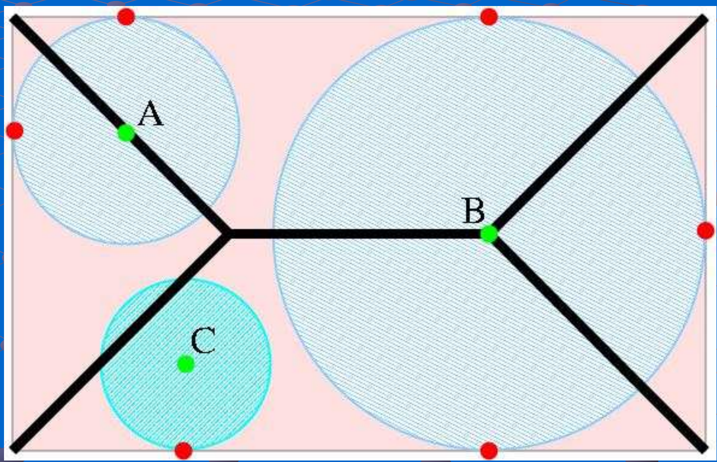
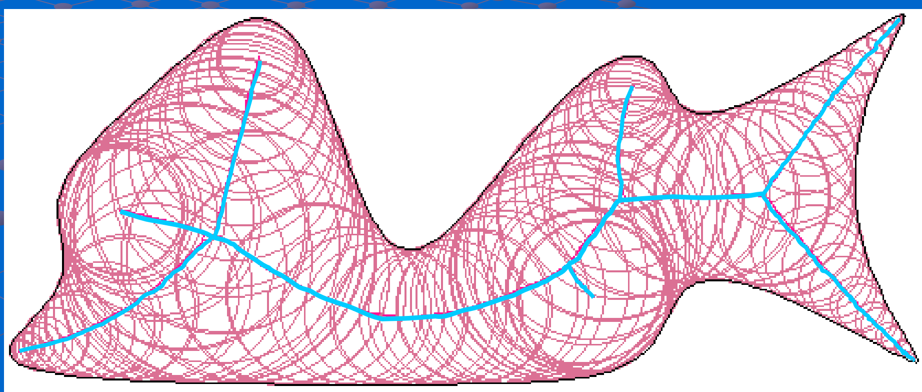
Skeleton

- *result of the Medial Axis Transform:* object points having at least two closest boundary points;
- *praire-fire analogy:* the boundary is set on fire and skeleton is formed by the loci where the fire fronts meet and quench each other;
- the locus of the centers of all the maximal inscribed hyper-spheres.

Nearest boundary points and inscribed hyper-spheres

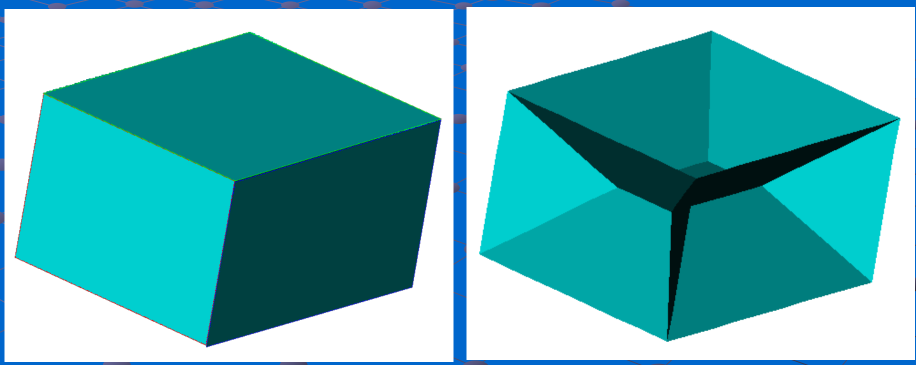


Object = union of the inscribed hyper-spheres



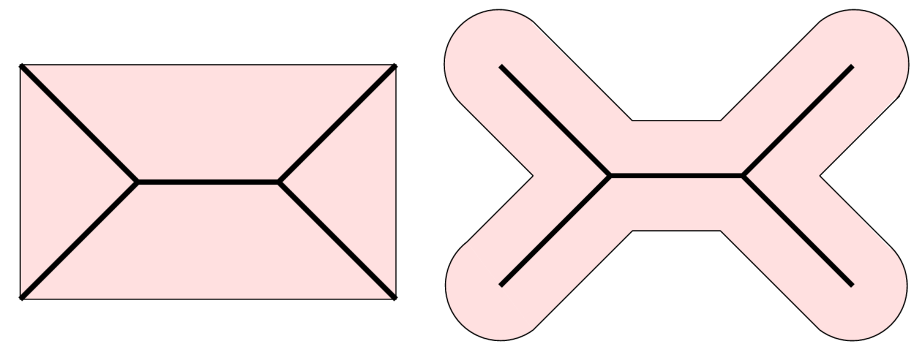
object boundary maximal inscribed disks centers

Skeleton in 3D



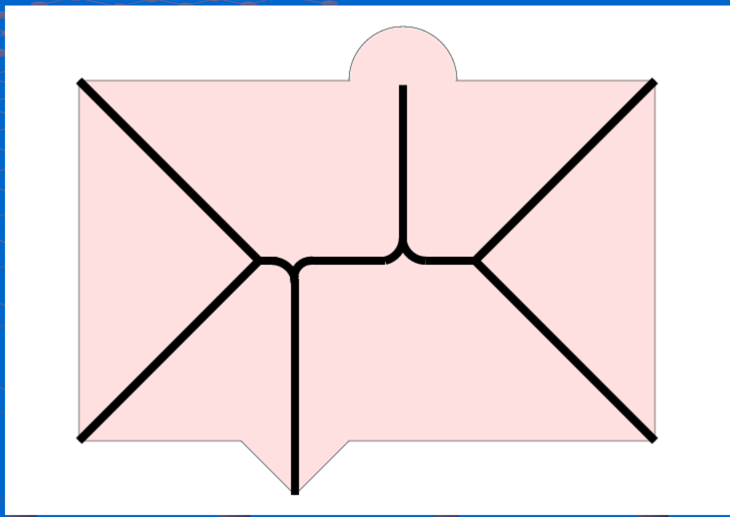
The skeleton in 3D generally contains surface patches (2D segments).

Uniqueness

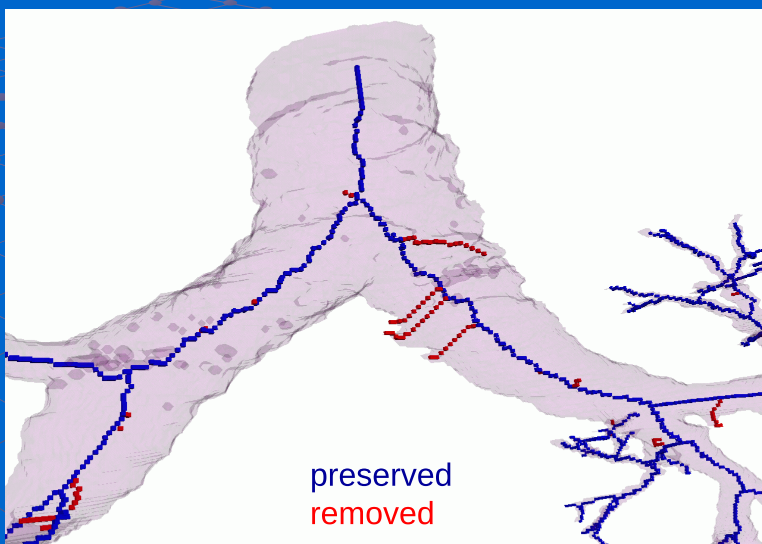


The same skeleton may belong to different elongated objects.

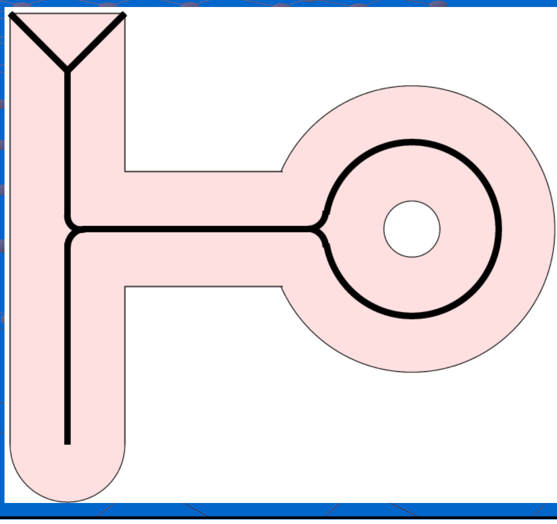
Stability



Pruning



Representing the topological structure



Properties

- represents
  - the general form of an object,
  - the topological structure of an object, and
  - local object symmetries.
- invariant to
  - translation,
  - rotation, and
  - (uniform) scale change.
- simplified and thin.

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- Skeletonization
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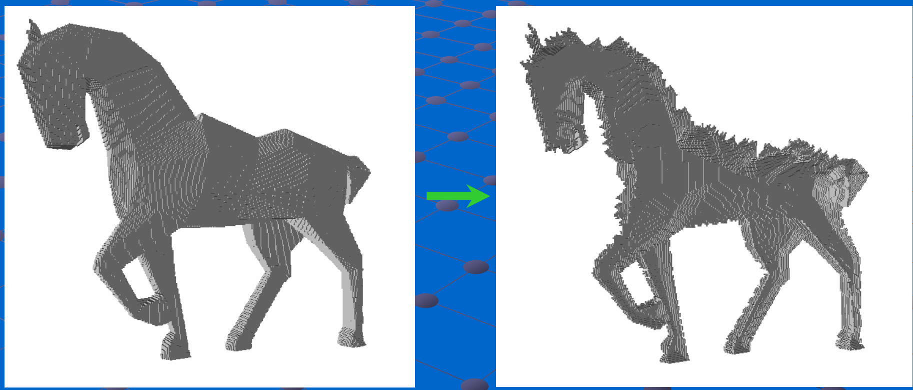
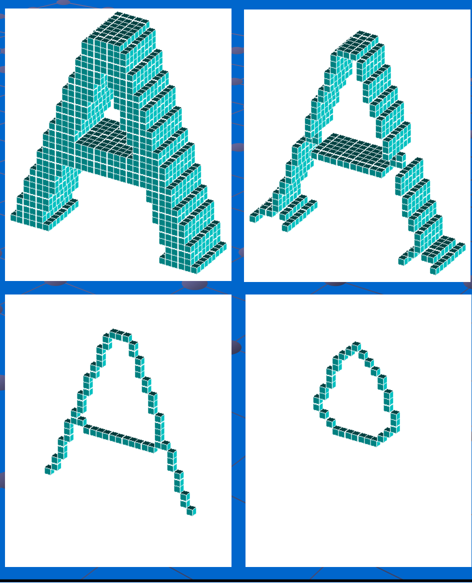
Skeletonization …

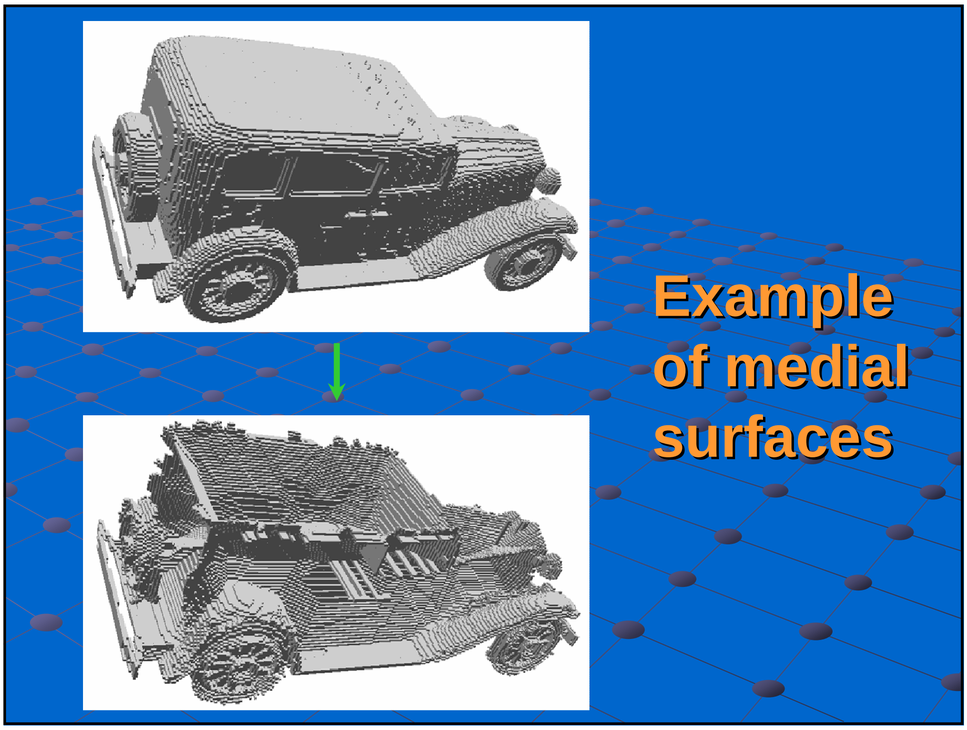
Skeletonization …

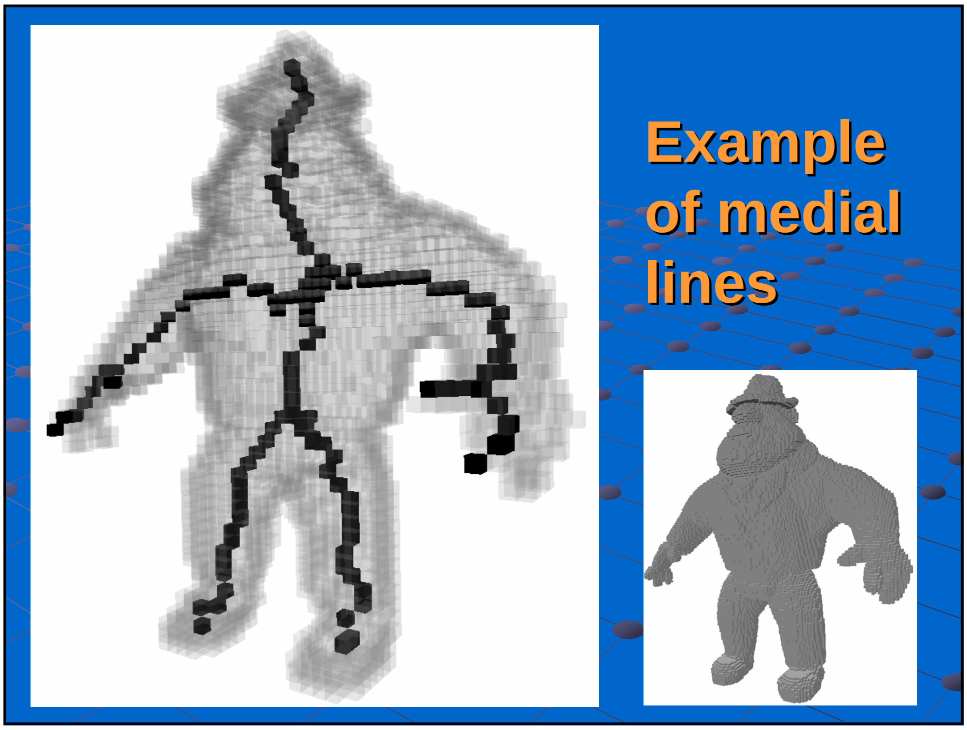


… means skeleton extraction from elongated binary objects.

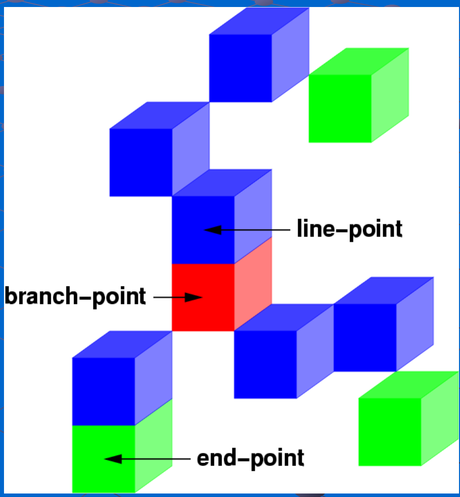
Skeleton-like descriptors in 3D



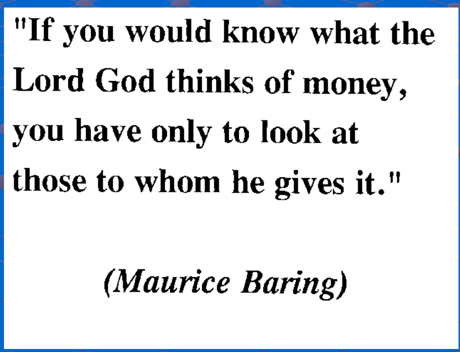

Example of medial surfaces


Example of medial lines

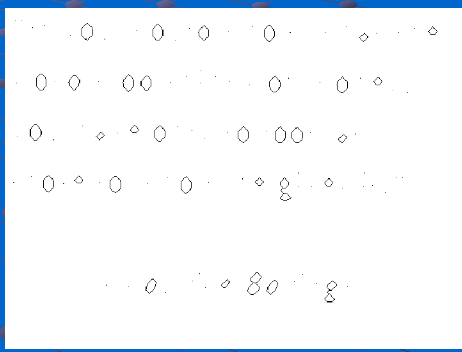
Skeletal points in 2D – points in 3D centerlines



Example of topological kernel

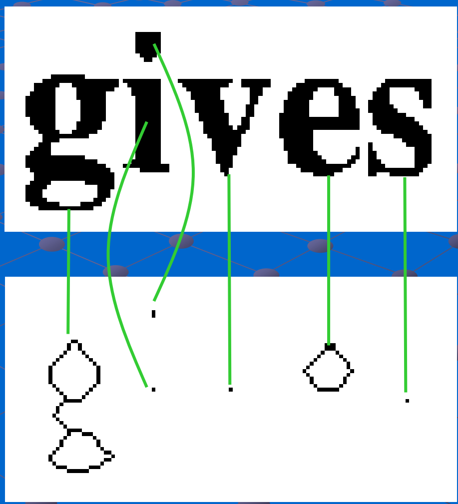


original image



topological kernel

Example of topological kernel



simply connected → an isolated point

multiply connected → closed curve

Skeletonization techniques

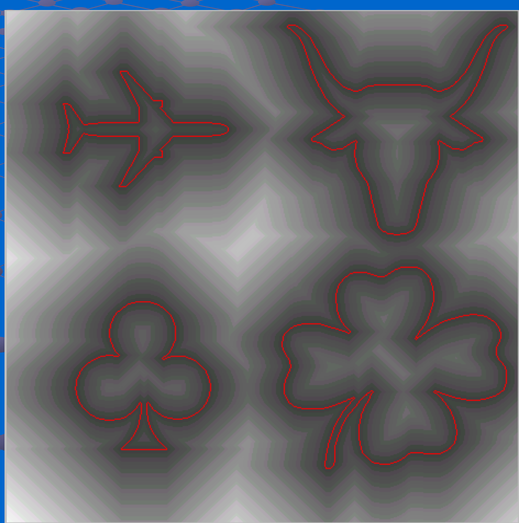
- distance transform
- Voronoi diagram
- thinning

Distance transform

Input:
Binary array *A* containing feature elements (1's) and non-feature elements (0's).

Output:
Distance map *B*: non-binary array containing the distance to the closest feature element.

Distance map



Distance transform using city-block (or 4) distance

4	3	2	1	2	3	4
3	2	1	0	1	2	3
2	1	0	1	0	1	2
2	1	0	1	1	0	1
1	0	1	2	2	1	0
1	0	1	2	3	2	1
0	1	2	3	4	3	2

	1	
1	0	1
	1	

Distance transform using chess-board (or 8) distance

2	2	1	1	1	2	2
2	1	1	0	1	1	2
2	1	0	1	0	1	1
1	1	0	1	1	0	1
1	0	1	1	1	1	0
1	0	1	2	2	1	1
0	1	1	2	2	2	2

1	1	1
1	0	1
1	1	1

Linear-time distance mapping

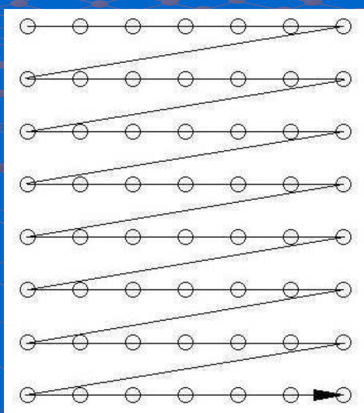
Input:
Binary array *A* containing feature elements (1's) and non-feature elements (0's).

Output:
Distance map *B*: non-binary array containing the distance to the closest feature element.

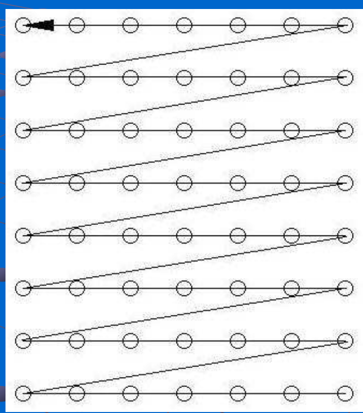
G. Borgefors (1984)

```
remark initialization
 for i=1 to n1 do
  for j=1 to n2 do
   if a(i,j)=1 then b(i,j)=0
   else             b(i,j)=∞
remark forward scan
 for i=1 to n1 do
  for j=1 to n2 do
   b(i,j)=min{
              b(i-1,j-1)+d2,
              b(i-1,j  )+d1,
              b(i-1,j+1)+d2,
              b(i  ,j-1)+d1,
              b(i  ,j  )
             }
remark backward scan
 for i=n1 downto 1 do
  for j=n2 downto 1 do
   b(i,j)=min{
              b(i  ,j  )   ,
              b(i  ,j+1)+d1,
              b(i+1,j-1)+d2,
              b(i+1,j  )+d1,
              b(i+1,j+1)+d2
             }
```

Linear-time distance mapping

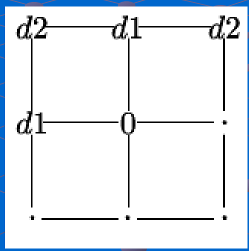


forward scan

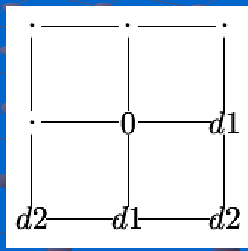


backward scan

Linear-time distance mapping



forward scan



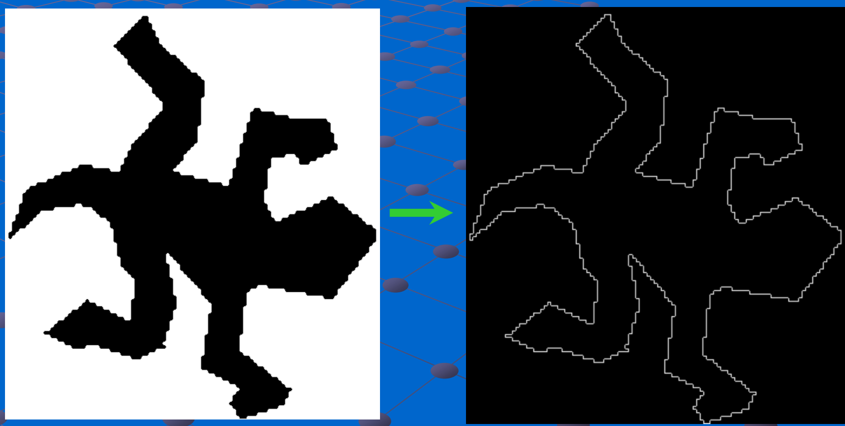
backward scan

best choice: $d1$=3, $d2$=4

Distance-based skeletonization

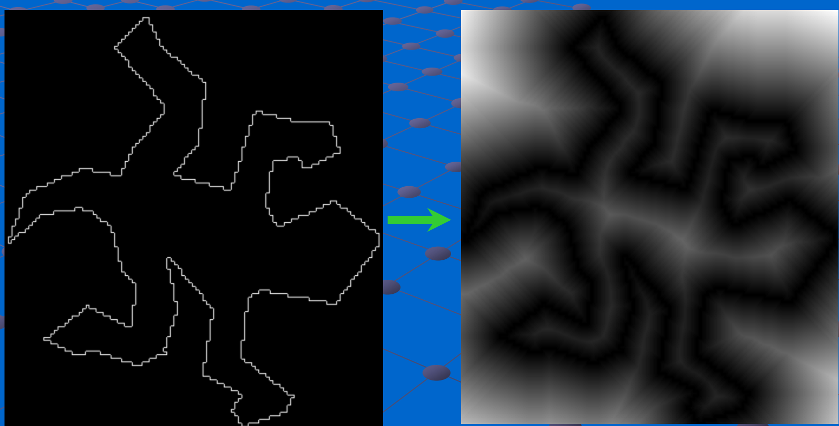
1. Border points (as feature elements) are extracted from the original binary image.
2. Distance transform is executed (i.e., distance map is generated).
3. The ridges (local extremas) are detected as skeletal points.

Distance-based skeletonization – step 1



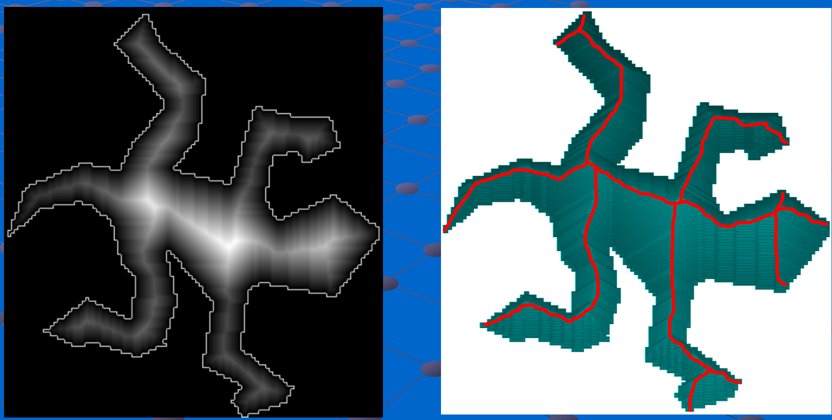
detecting border points

Distance-based skeletonization – step 2

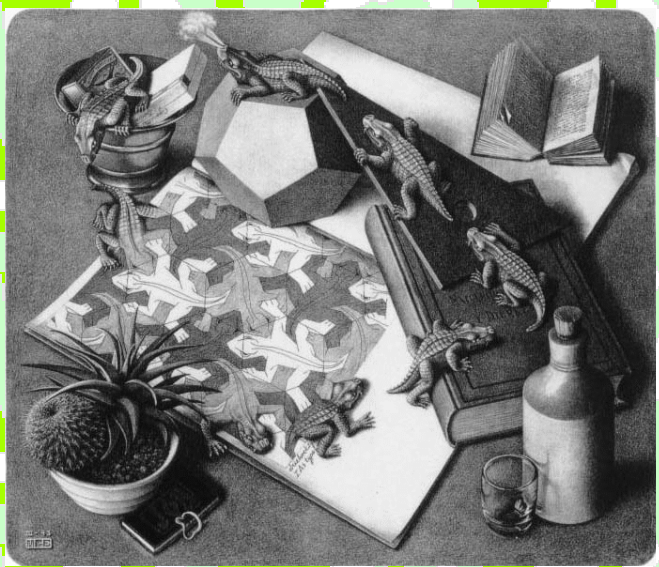


distance mapping

Distance-based skeletonization – step 3



detecting ridges (local extremas)



M.C. Escher: Reptiles

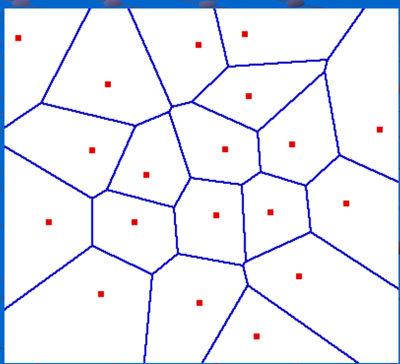
Skeletonization techniques

- distance transform
- Voronoi diagram
- thinning

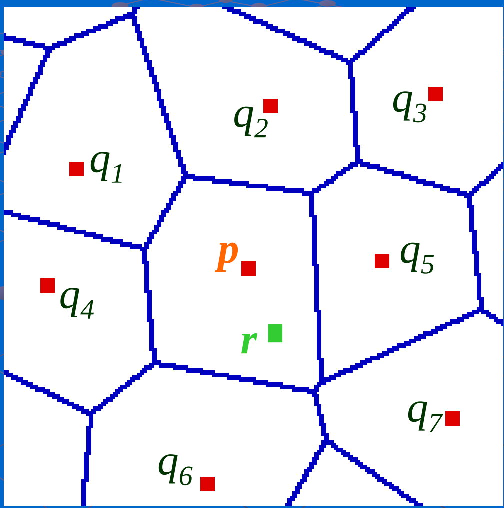
Voronoi diagram

Input:
Set of points (generating poins)

Output:
the partition of the space into cells so that each cell contains exactly one generating point and the locus of all points which are closer to this generating point than to others.

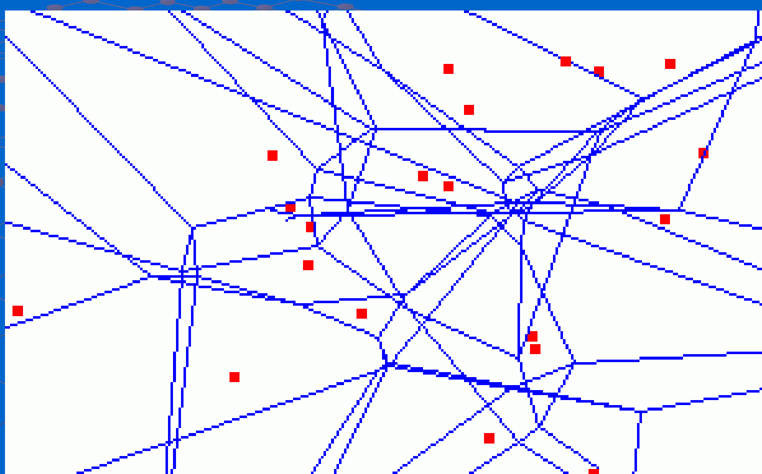


Voronoi diagram



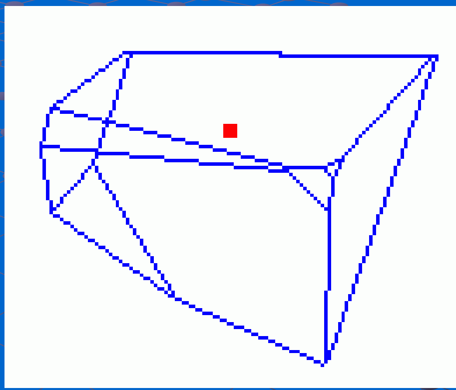
$$d(r,p) \le d(r,q_i)$$
$$(i = 1,2,\ldots)$$

Voronoi diagram in 3D



Voronoi diagram of 20 generating points

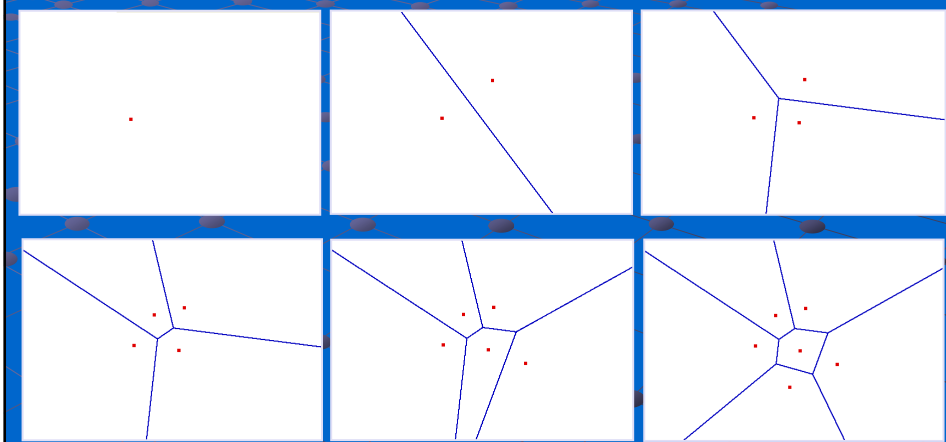
Voronoi diagram in 3D



a cell (convex polyhedron) of that Voronoi diagram

Incremental construction

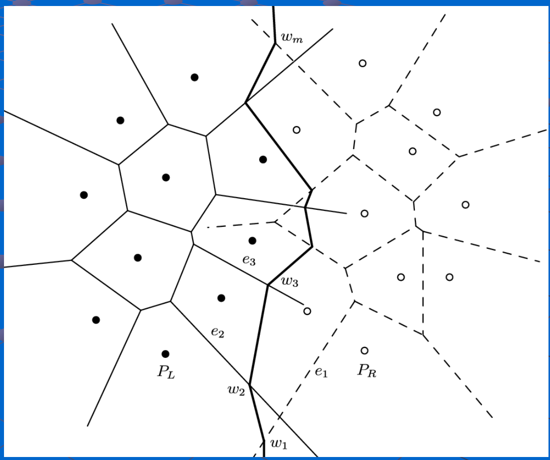
$O(n)$



Divide and conquer

$O(n \cdot \log n)$

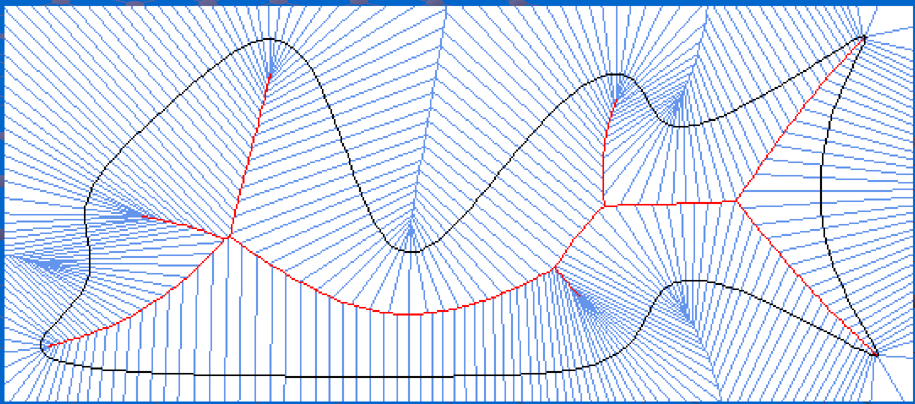
left diagram



right diagram

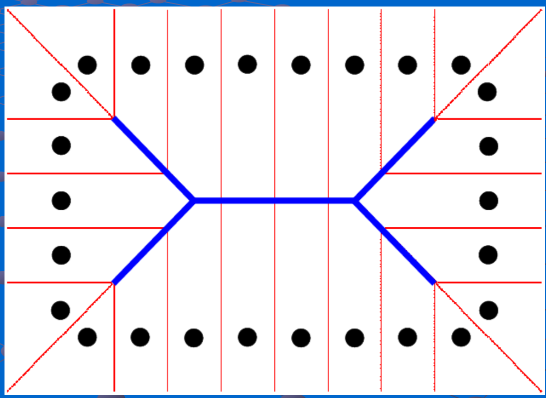
merging

Voronoi diagram - skeleton



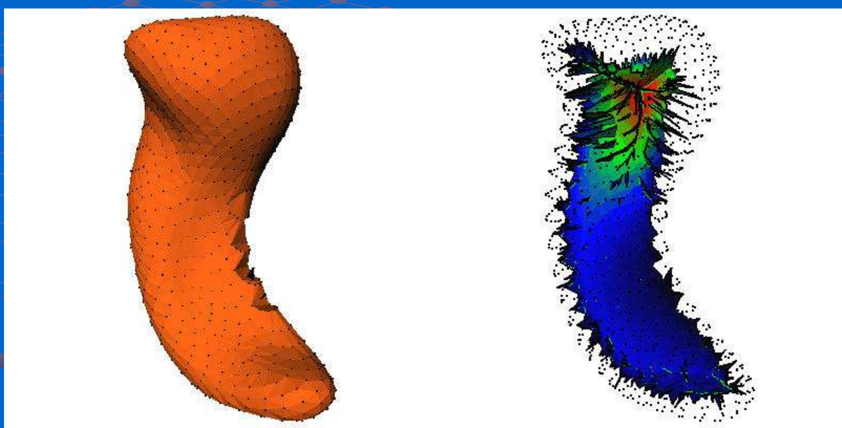
set of generating points = sampled boundary

Voronoi diagram - skeleton



If the density of boundary points goes to infinity, then the corresponding Voronoi diagram converges to the skeleton.

Voronoi skeleton



original 3D object

Voronoi skeleton

M. Styner (UNC, Chapel Hill)

Skeletonization techniques

- distance transform
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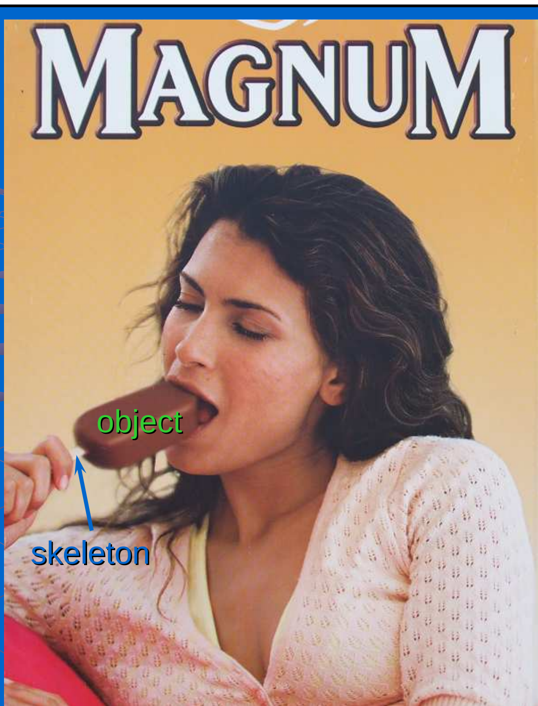


„Thinning”

Iterative object reduction

Matryoshka:
Russian nesting wooden doll.

original object

reduced structure

Thinning algorithms

repeat

remove „*deletable*" points
from the actual binary image

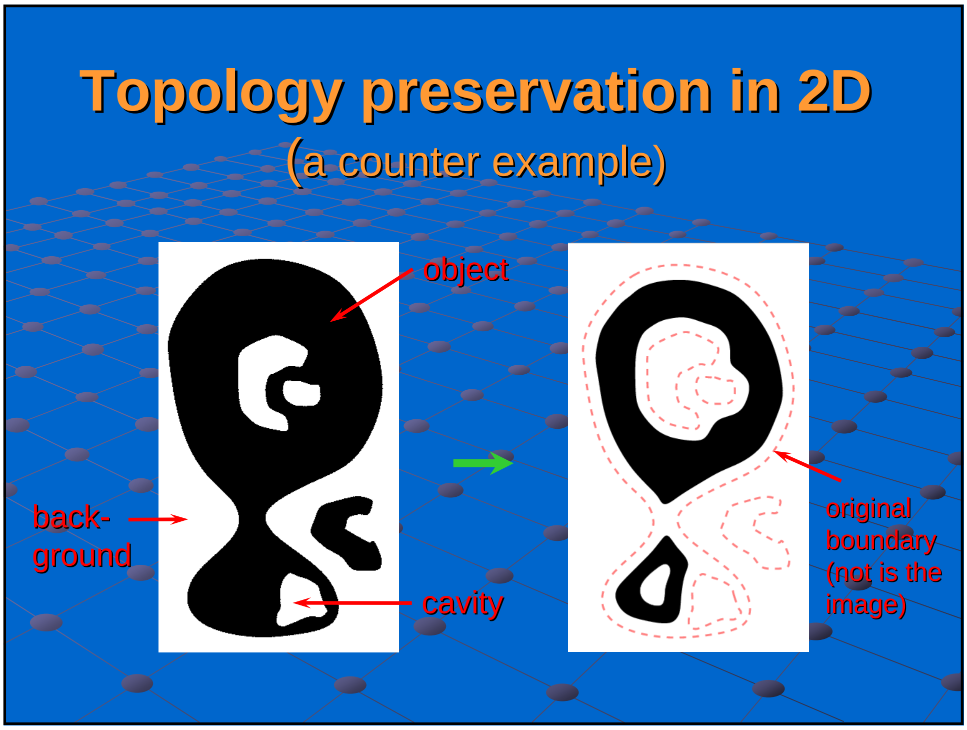
until no points are deleted

$\}$ one iteration step

<u>degrees of freedom:</u>

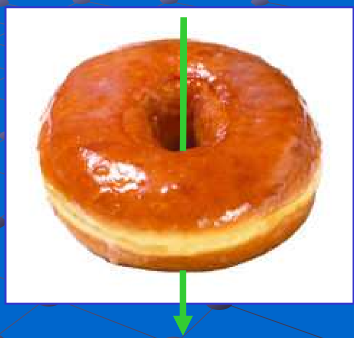
– which points are regarded as „*deletable*" ?
– how to organize one iteration step?


One iteration step


Topology preservation in 2D
(a counter example)
object
back-
ground
cavity
original
boundary
(not is the
image)

Topology in 3D

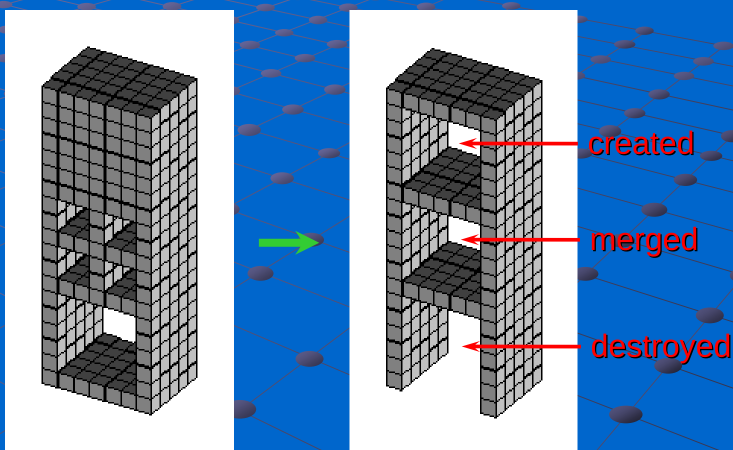
hole - a new concept



”A topologist is a man who does not know the difference between a coffee cup and a doughnut.”

Topology preservation in 3D

(a counter example)



Shape preservation

"If you would know what the Lord God thinks of money, you have only to look at those to whom he gives it."

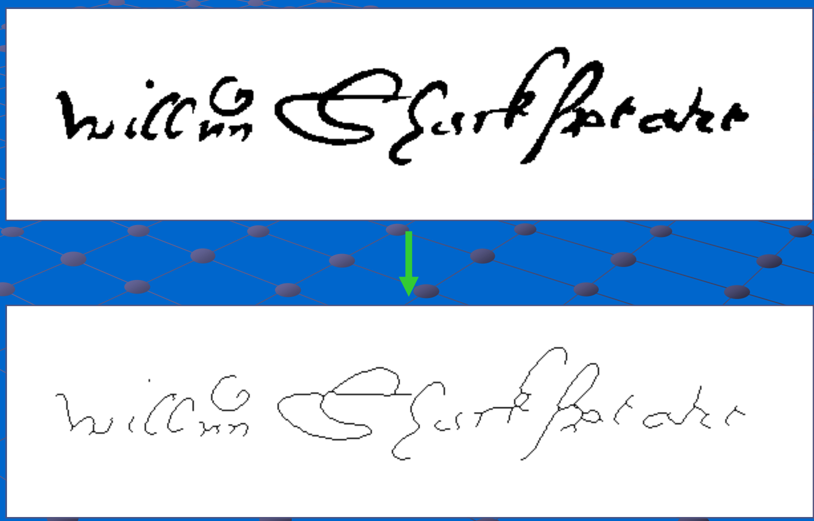
(Maurice Baring)

Shape preservation

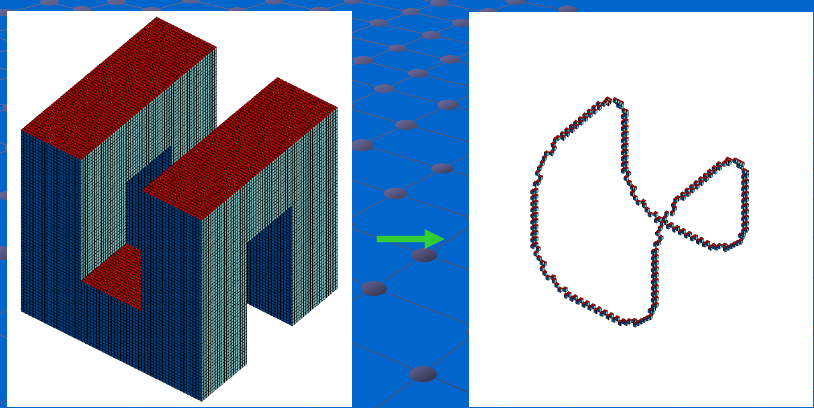
"If you would know what the Lord God thinks of money, you have only to look at those to whom he gives it."

(Maurice Baring)

Example of 2D thinning



Example of 3D thinning



original object

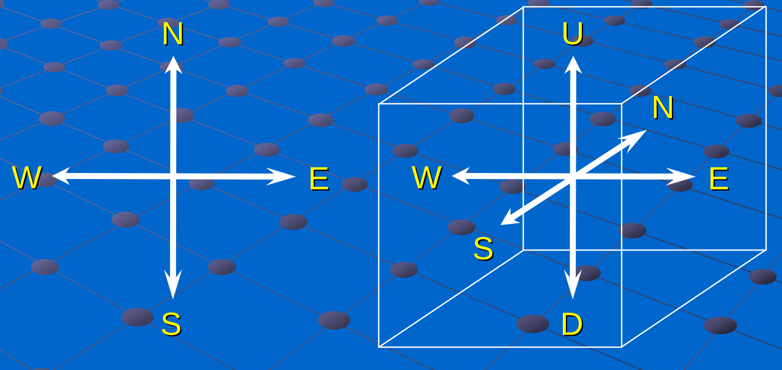
centerline

I prefer thinning since it …

- allows direct centerline extraction in 3D
- makes easy implementation possible
- takes the least computational costs
- can be executed in parallel.

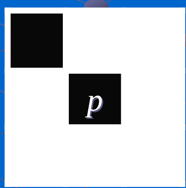
Some concepts …

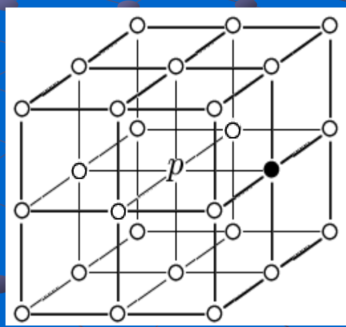
main *directions*:



Some concepts …

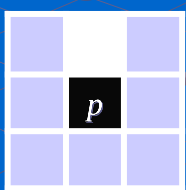
p is a **line-end point**
if it is adjacent to just one object point

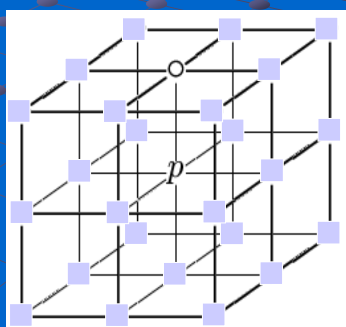




Some concepts …

p is a **border point** if it is (4/6-)adjacent
to at least one non-object point





don't care (either 0 or 1)

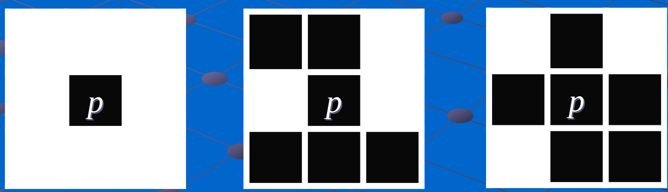
border point of type N

border point of type U

Some concepts …

An object-point is **simple** if its deletion doesn't alter the topology of the picture.

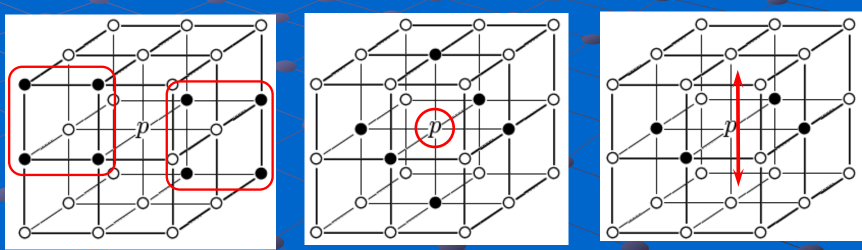
Examples of **non-simple** points in 2D pictures:



deleting an object | splitting an object | creating a cavity

Some concepts …

Examples of **non-simple** points in 3D pictures:



splitting an object | creating a cavity | creating a hole

Some concepts …

*Simple*ness is a local property:
It depends on the 3x3 / 3x3x3 neighborhood
of the point in question.

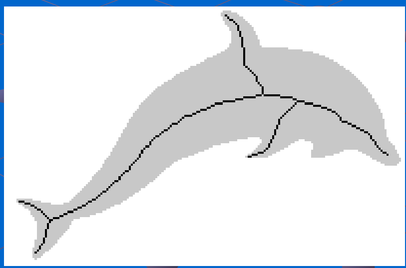
↓

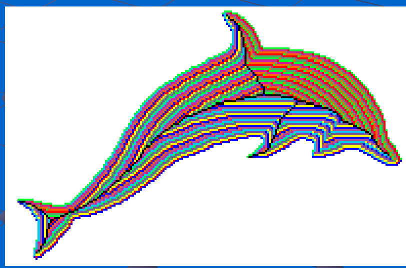
It can be decided by using a precalculated
LUT (look-up table) of size 128 bit / 8 Mbyte.

A 4-subiteration parallel 2D thinning algorithm (*Rosenfeld, 1975*)

repeat
  for each directions N,E,S, and W **do**
    delete object point if it is
      - a *border* point
        according to the actual direction,
      - *not* a *line-end* point, and
      - *simple*
until no points are deleted

A 4-subiteration parallel 2D thinning algorithm
(Rosenfeld, 1975)

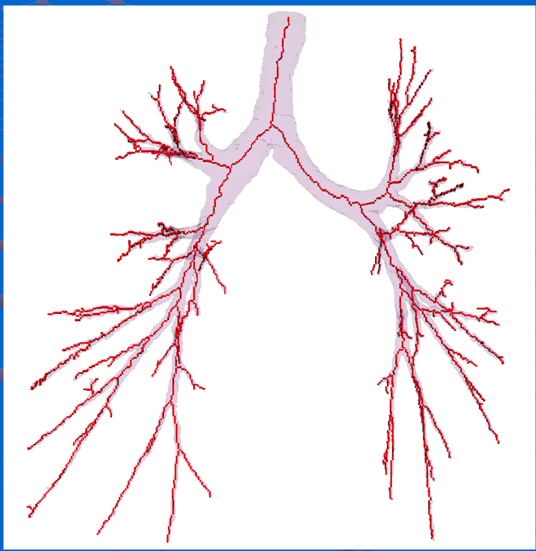




A sequential 3D thinning algorithm
(Palágyi et al., 2001)

repeat
  for each direction U,N,E,S,W, and D **do**
    - mark object point p if it is
      - *border* according to the actual direction,
      - *not line-end* points, and
      - *simple*
    - **for** each marked point q **do**
      delete q if it is
        - not a *line-end* point, and
        - *simple* in the actual picture
until no points are deleted

A sequential 3D thinning algorithm
(*Palágyi et al., 2001*)



Requirements

- Geometrical:
  The skeleton must be in the middle of the original object and must be invariant to translation, rotation, and scale change.
- Topological:
  The skeleton must retain the topology of the original object.

Comparison

method	geometrical	topological
distance-based	yes	no
Voronoi-based	yes	yes
thinning	no	yes

Syllabus

- Shape
- Shape features
- Skeleton
- Skeletonization
- Applications

Applications in 2D

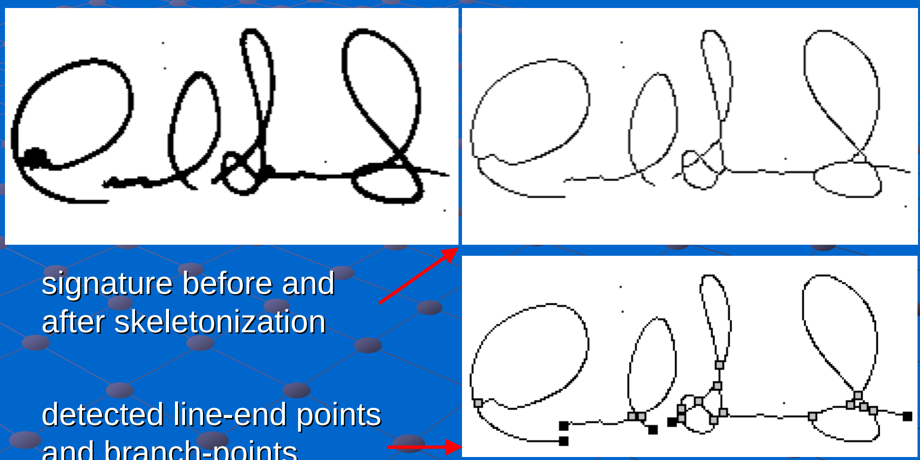
- „exotic” character recognition
- recognition of handwritten text
- signature verification
- fingerprint and palmprint recognition
- raster-to-vector-conversion
- …

Exotic character recognition

characters of a Japanese signature

K. Ueda

Signature verification

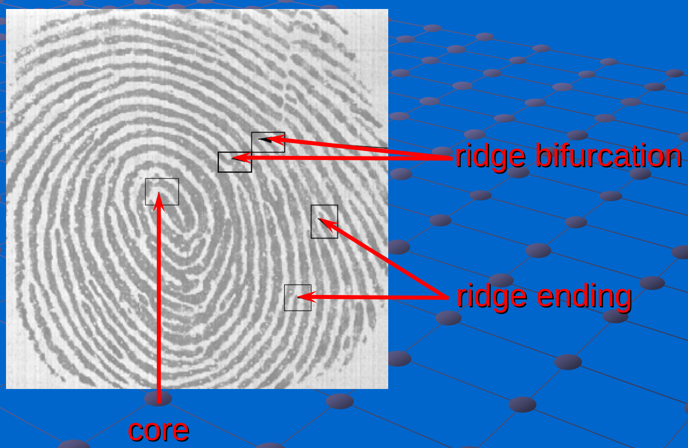


signature before and after skeletonization

detected line-end points and branch-points

L.C. Bastos et al.

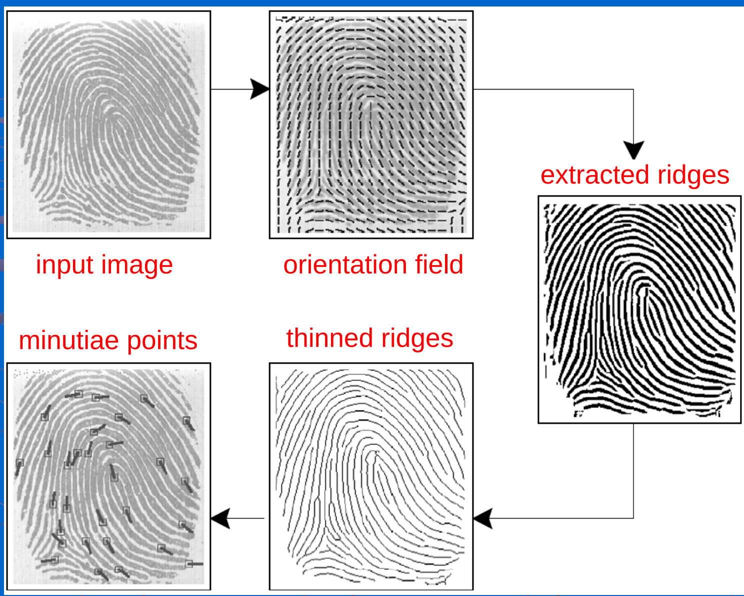
Fingerprint verification



features in fingerprints

A. Ross

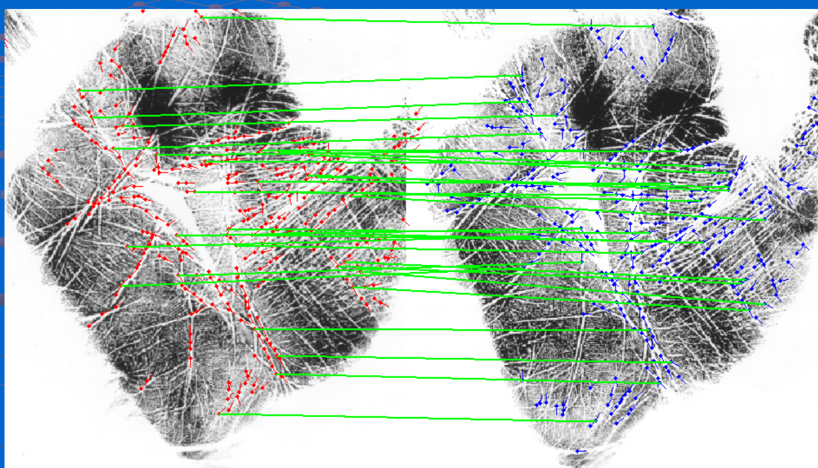
Fingerprint verification



the process

A. Ross

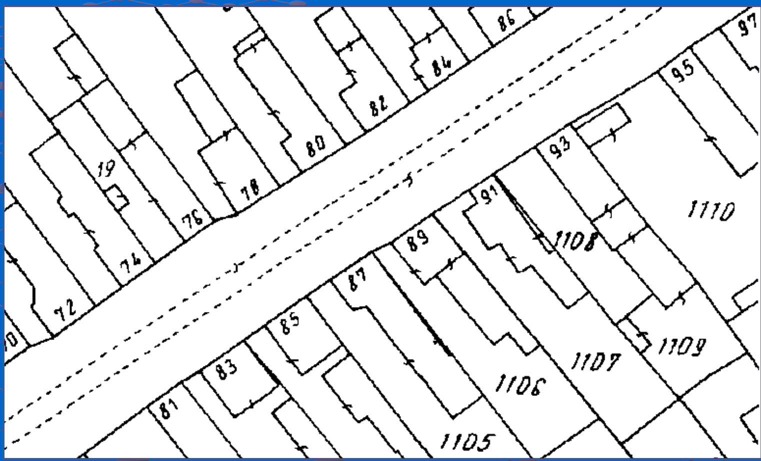
Palmprint verification



matching extracted features

N. Duta

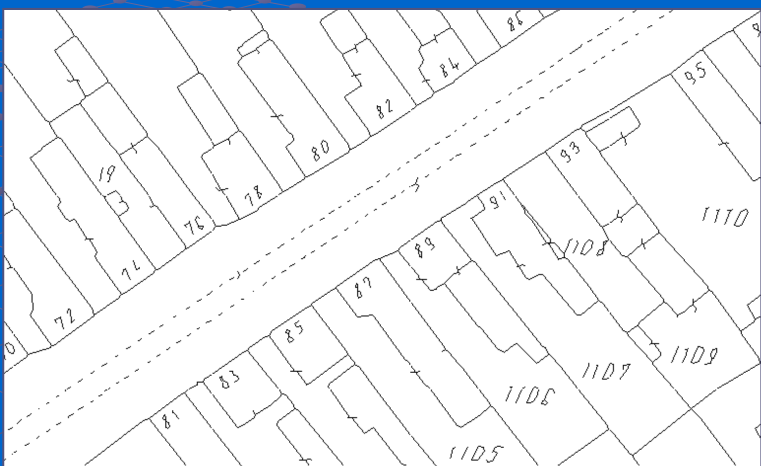
Raster-to-vector conversion



scanned map

Katona E.

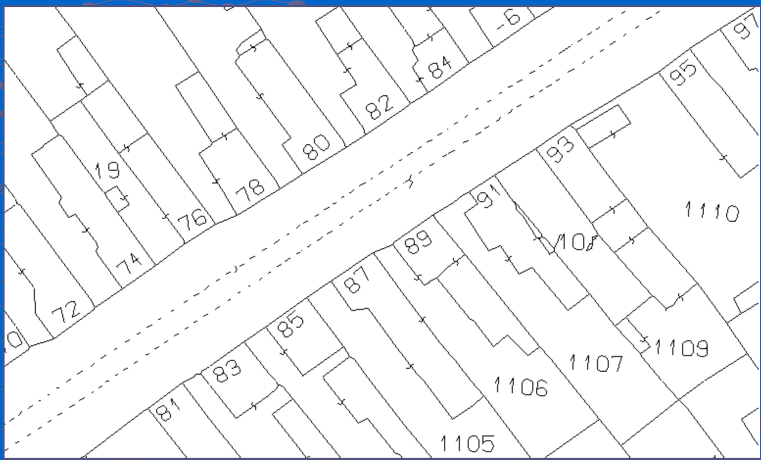
Raster-to-vector conversion



„raw” vector image after skeletonization

Katona E.

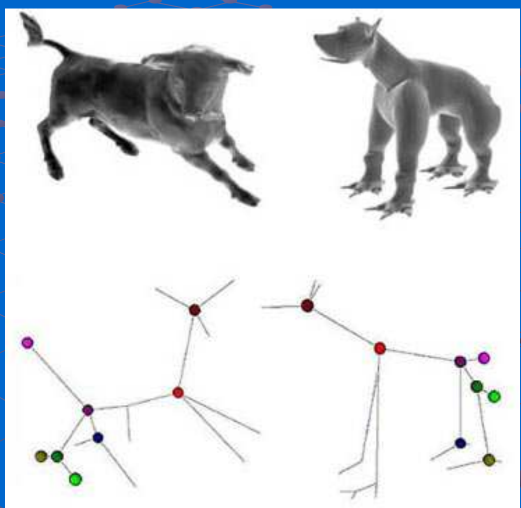
Raster-to-vector conversion



corrected vector image

Katona E.

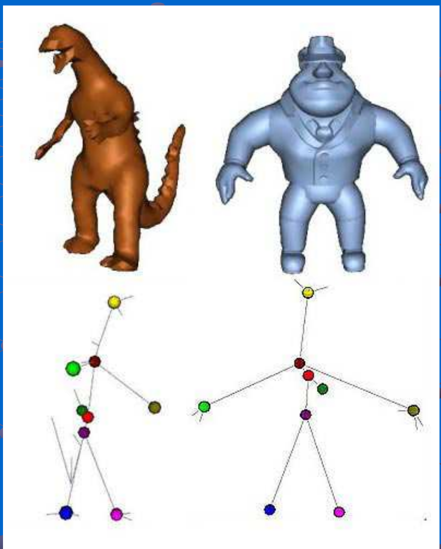
Shape matching and retrieval



- skeletal graph construction
- graph matching

Sundar et al., 2003

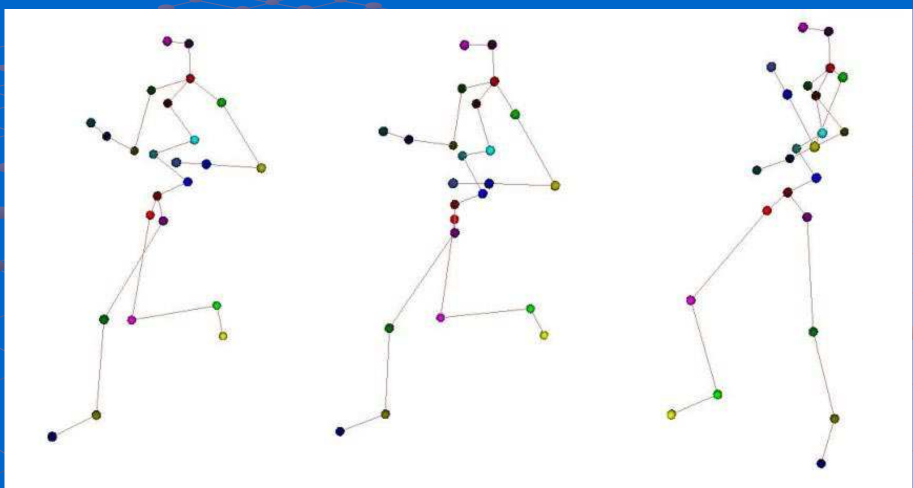
Shape matching and retrieval



- skeletal graph construction
- graph matching

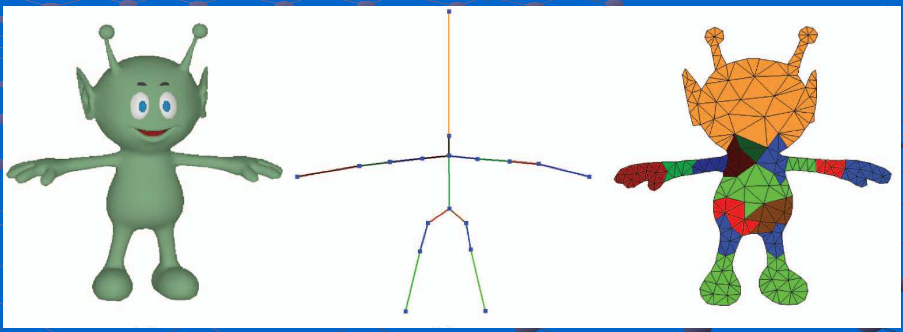
Sundar et al., 2003

Shape matching and retrieval



Sundar et al., 2003

Shape deformation



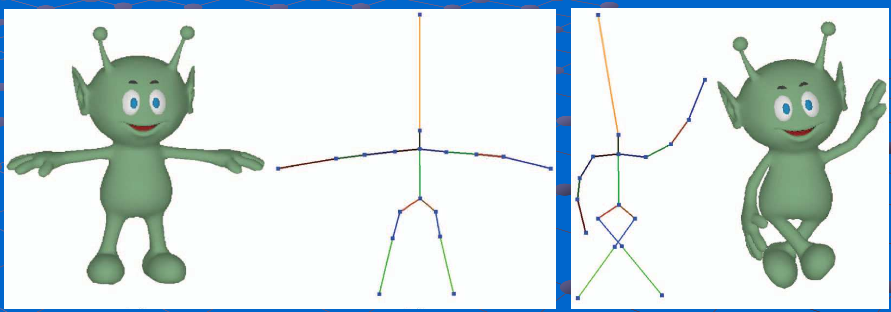
object

partitioned skeleton

skeleton control domain

Yan et al., 2008

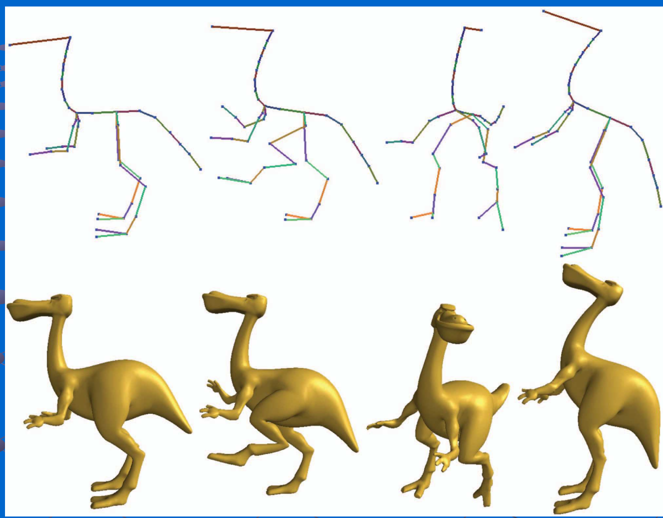
Shape deformation



deformed skeleton and object

Yan et al., 2008

Shape deformation

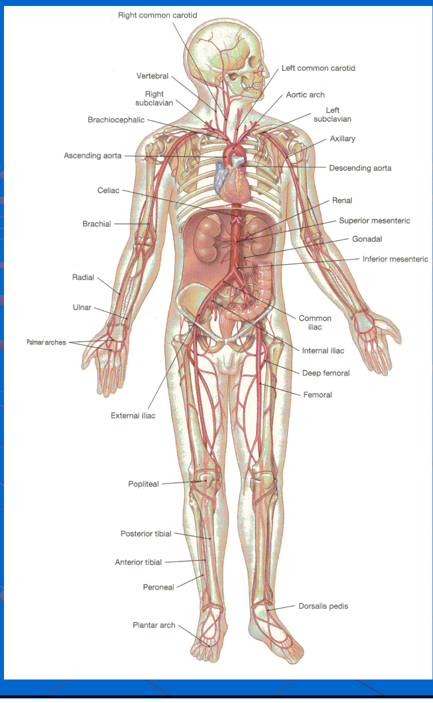


deformed skeletons and objects

Yan et al., 2008

Medical applications in 3D

There are some frequently used 3D medical scanners (e.g., CT, MR, SPECT, PET), therefore, applications in medical image processing are mentioned.

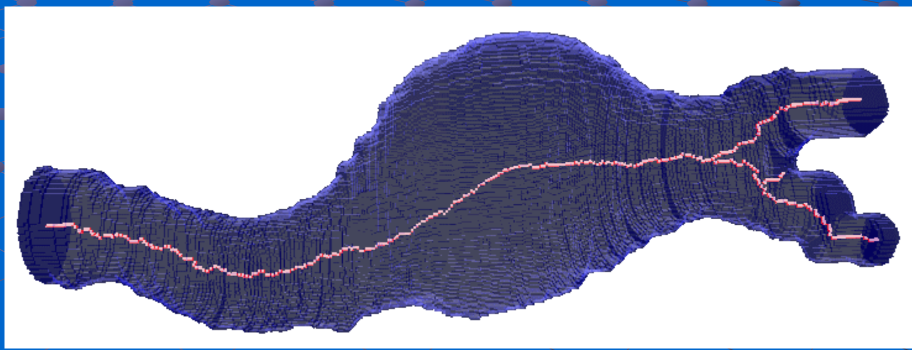


There are a lots of tubular structures (e.g., blood vessels, airways) in the human body, therefore, centerline extraction is fairly important.

Applications based on centerline extraction

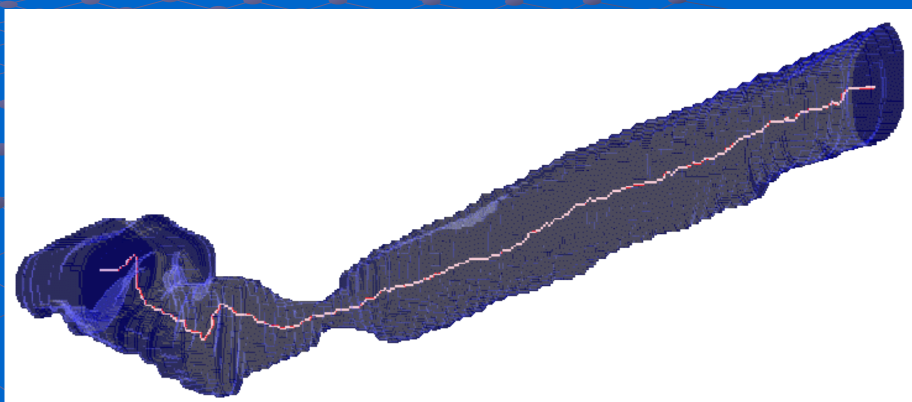
E. Sorantin et al.

Blood vessel
(infra-renal aortic aneurysms)



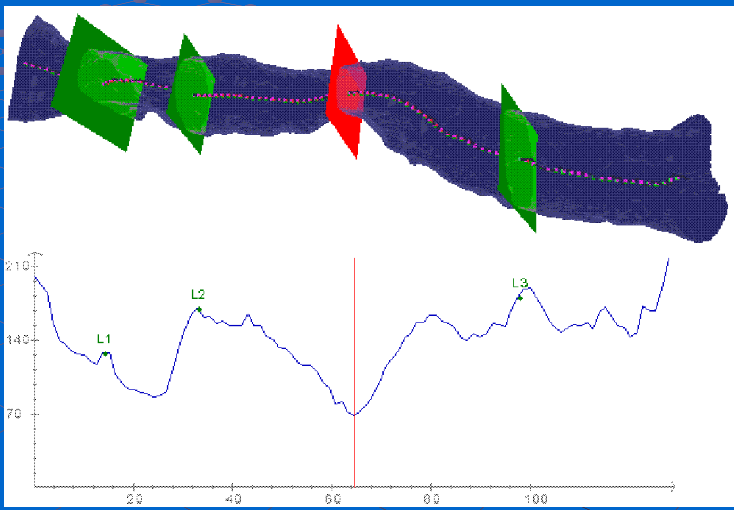
E. Sorantin et al.

Airway
(trachealstenosis)



E. Sorantin et al.

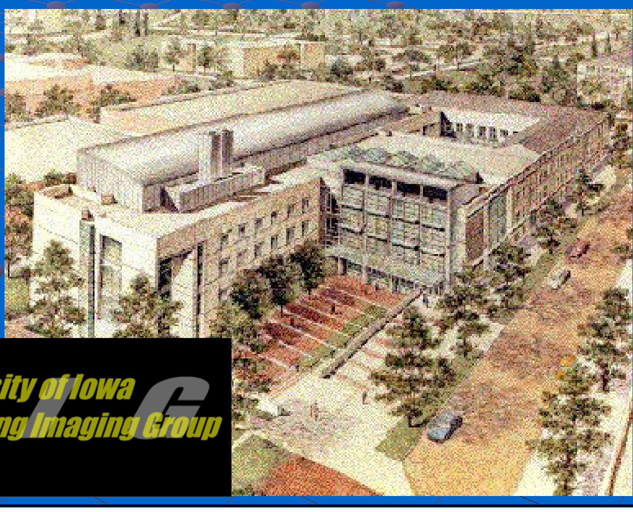
Airway (trachealstenosis)



E. Sorantin et al.

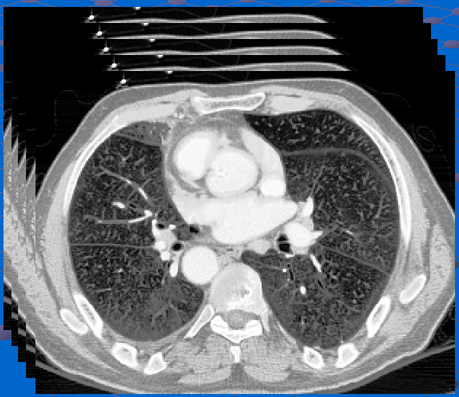
Quantitative analysis of intrathoracic airway trees

Kálmán Palágyi
Juerg Tschirren
Milan Sonka
Eric A. Hoffman



The University of Iowa
College of Engineering Imaging Group

Images



Multi-detector
Row Spiral CT

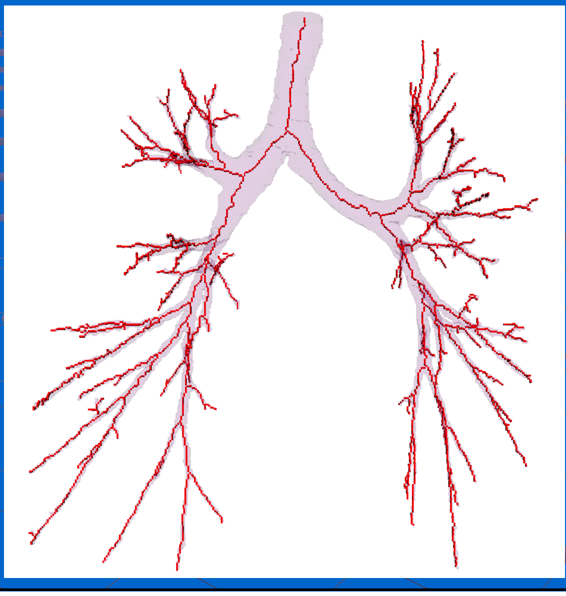
512 x 512 voxels

500 – 600 slices

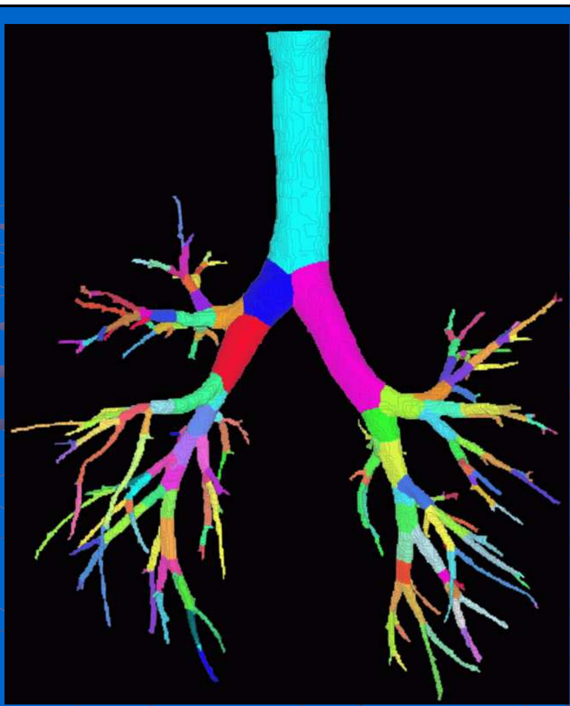
0.65 x 0.65 x 0.6 mm^3
(almost isotropic)

Lung segmentation

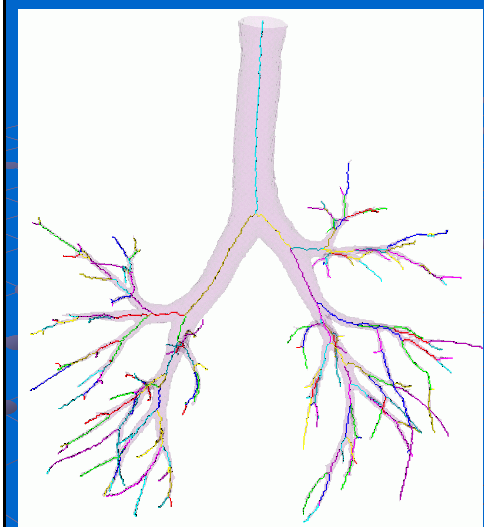
Centerlines



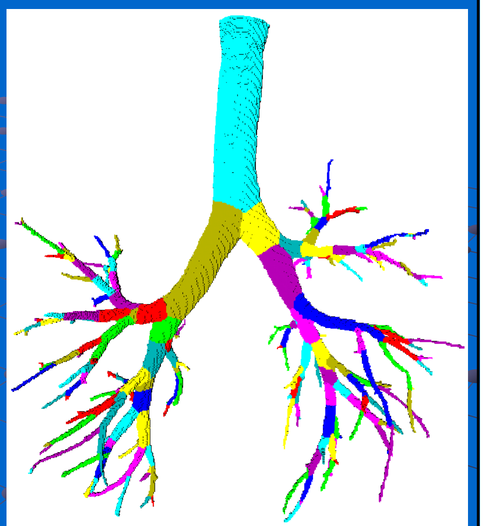
detected branch-points



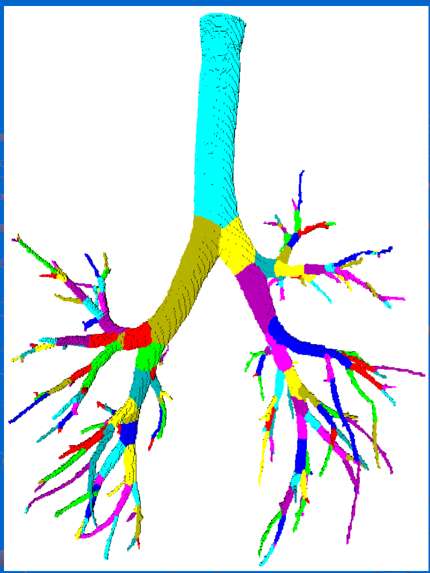
Branch partitioning



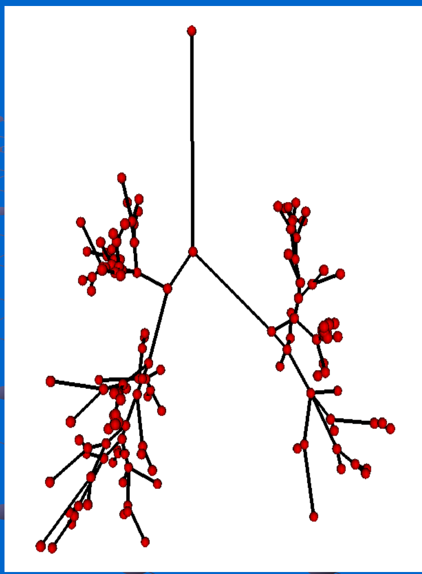
centerline labeling



label propagation



labeled tree

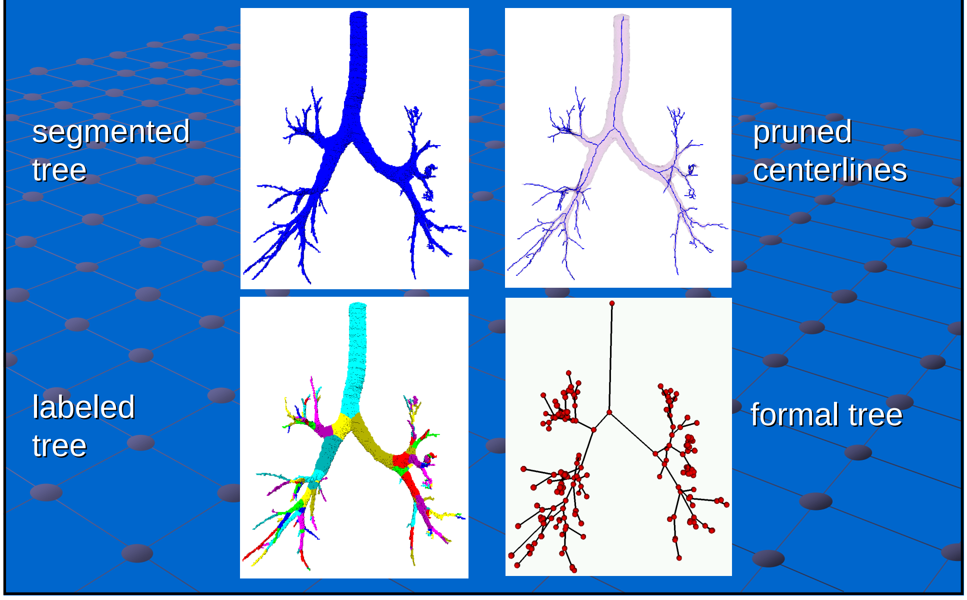


formal tree (in XML)

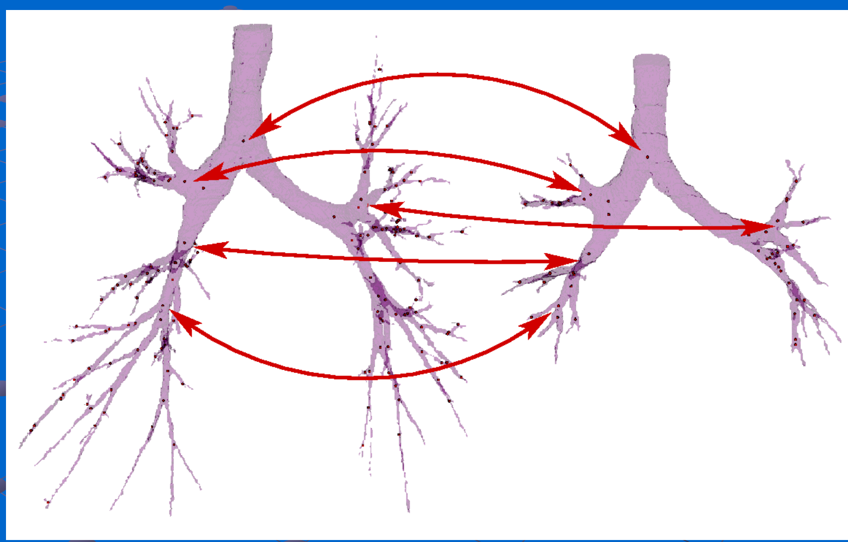
Quantitative indices for tree branches

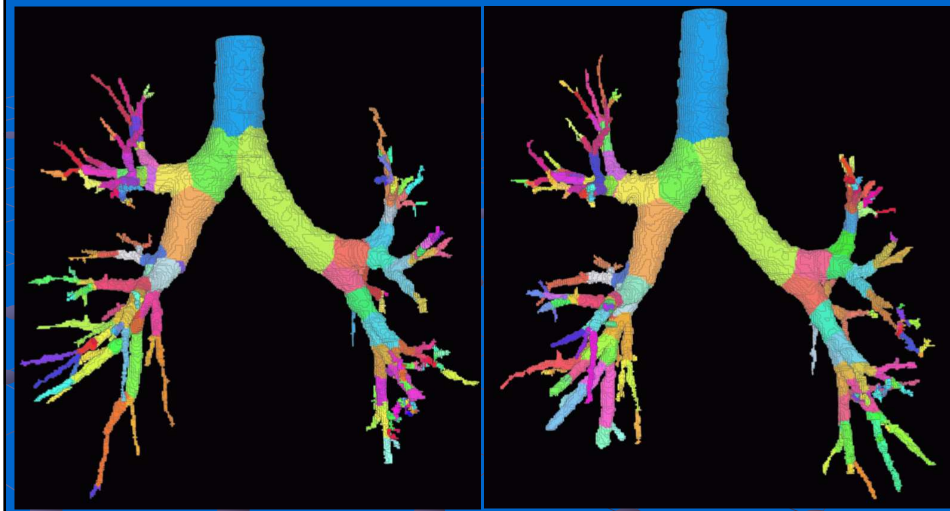
- length (Euclidean distance between the parent and the child branch points)
- volume (volume of all voxels belonging to the branch)
- surface area (surface area of all boundary voxels belonging to the branch)
- average diameter (assuming cylindric segments)

The entire process



Matching





FRC

TLC

Anatomical labeling