Introduction to Image Registration

Sven Lončarić

Image Processing Group
Faculty of Electrical Engineering and Computing
University of Zagreb, Croatia
http://ipg.zesoi.fer.hr


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Zagreb: SSIP 2000 Venue

Outline of Presentation

- Introduction
- Classification of image registration methods
- Overview of geometric transformations
- Overview of registration algorithms
- Conclusion

Introduction

- Image registration (also called image matching) is an important problem in image analysis with many applications:
  - Several images of the same object are taken using different imaging modality
  - Several images of the same object are taken at different time instants
  - It is necessary to compare two objects
  - It is desired to match an image to a model (e.g. digital atlas)

The Problem

- The problem of image registration is to determine an unknown geometric transformation that maps one image into another (to a certain degree of accuracy)
  - In other words, after registration problem is solved, for each pixel in the first image we know the corresponding pixel in the second image
- This assumes that the images are similar in the sense that both images contain the same (or similar) object, which may be rotated, translated, or elastically deformed

The Problem

- In medical applications, image registration is usually done for two-dimensional and three-dimensional images
- In general, registration problem can be solved in any number of spatial or temporal dimensions

Motivation

- When two images are registered it is possible to:
  - Analyze (detect) differences between the images (e.g. images taken at two different time instants or difference between the template and a tested product in visual inspection)
  - Combine information contained in multiple images into a single image (image fusion) with the goal of easier interpretation by humans (e.g. in radiology it is possible to do multimodality image registration – MR to CT, etc.)

Information Integration

- Information integration has the goal of combining several pieces of information into a single one
  - E.g. merge several images into a single one
- In the context of image processing this is called image fusion
- There are several probabilistic theories for information integration such as:
  - Bayesian approach
  - Dempster-Schefer theory

Applications: Remote Sensing

- In remote sensing (e.g. meteorology) the same geographical area may be imaged in various spectral ranges
- In management of urban areas it is possible to take images of an urban area in regular time intervals and detect changes (e.g. new buildings)

Applications: Medicine

- Biomedicine is an important application area
- Developments in medical imaging resulted in powerful imaging modalities providing information about anatomy and function of the human body:
  - Computed tomography (CT)
  - Magnetic resonance (MR)
  - Ultrasound
  - Positron emission tomography (PET)
  - Single photon emission computed tomography (SPECT)
  - Gamma camera imaging
  - X-ray imaging

Applications: Medicine

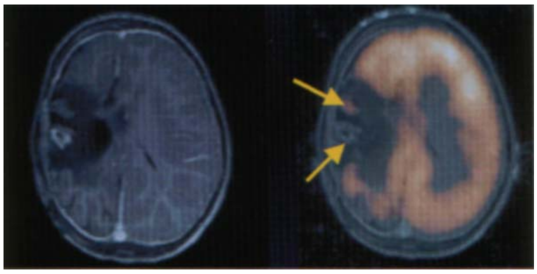
- Medical image registration is required for:
  - Use of different imaging modalities (e.g. MR/PET, MR/CT)
  - Progressive disease tracking (imaging in regular time intervals and detection of changes, e.g. for tumor treatment evaluation)
  - In computer assisted surgery (e.g. in neurosurgery preoperative MR images may be registered with intraoperative MR images for surgical navigation)
  - Matching of patient images to a model (e.g. for atlas-guided image analysis)

Example: Hand Registration

- X-ray image (anatomical information)
- Nuclear medicine image (functional information)
- After registration, hand image obtained by nuclear medicine imaging is pseudocolored and superimposed on the gray scale X-ray hand image
- Red color corresponds to the largest isotope concentration

Example: Brain Registration

- MR image showing anatomy (left), PET FDG image showing function superimposed on MR image (right)



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Classification of Methods

- Registration methods can be classified with respect to several different criteria such as:
  - Dimensionality of images that are registered (2-D, 3-D, 4-D methods)
  - Image features being matched (extrinsic and intrinsic methods)
  - Mechanism of interaction with the user
  - Type of geometric transformation used for registration

Dimensionality

- We can register 2-D, 3-D, or 4-D images
- 3-D registration can be done for two 3-D images or for a temporal sequence of 2-D images (video)
- If we want to register a sequence of 3-D images this represents a 4-D registration problem
- There is also a problem of 2-D image (perspective projection of 3-D space) to 3-D image registration
  - In this case it is necessary to determine the view parameters so that the obtained perspective transformation of 3-D image matches the 2-D image

Image Features

- This classification is motivated by the type of image features used for image registration:
  - Extrinsic methods (external objects or markers are used as reference points for registration)
  - Intrinsic methods (registration is based on pixel values - no external objects are used)

Extrinsic Methods

- Extrinsic methods use artificial external objects (markers) attached to the object to be registered
- Markers are detected in both images and used for registration
  - Example: For brain image registration skin markers or stereotactic frames may be used
- Disadvantage: Registration is based on external markers so accuracy depends on the accuracy of marker detection (segmentation is required)

Intrinsic Methods

- Intrinsic registration methods do not use artificial external objects
- Intrinsic methods use:
  - anatomical landmarks (points, contours, or surfaces), or
  - pixel values (intensity-based methods)
- Anatomical landmarks must be detected and this represents a disadvantage (possibility of error)
- Intensity-based methods have advantage of relying only on pixel values without the need for detection of special landmarks

User Interaction

- With respect to user interaction, registration methods can be divided into:
  - Interactive (require user interaction to define the geometric transformation for registration):
  - Semi-automatic (user interaction is only required for initialization, guidance, or stopping the registration procedure)
  - Automatic (do not require any user interaction)

Geometric Transformations

- This classification is based on the type of transformation used for registration:
  - Rigid registration: distance between any two object points is preserved (rotation, translation)
  - Affine transformation: A line is mapped into a line, parallelism between lines is preserved
  - Projection transformation (e.g. perspective projection) is like affine, but it does not preserve parallelism of lines
  - Elastic transformation: line is mapped into a curve

Geometric Transformations

- Rigid transformations are a subset of affine transformations
- Affine transformations are a subset of projective transformations
- Projective transformations are a subset of elastic transformations

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Overview of Transformations

- In the next several slides we present an overview of the basic geometric transformations:
  - Rigid transformations
  - Scaling transformations
  - Affine transformations
  - Projective transformations
  - Perspective transformations
  - Elastic transformations

Rigid Transformations

- A rigid transformation of vector **x** = [x y z]^T consists translation and rotation:

  x' = Rx + t

  where **t** = [t_x t_y t_z]^T is translation vector, **R** is a 3x3 orthogonal rotation matrix (α, β, and γ are rigid body rotation angles around z, y, and x axes)

$$R = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}$$

Scaling Transformations

- The simplest affine transformations are those that only include scaling, while the rest of the transformation is rigid:

  x' = RSx + t

  where **S** = diag(s_x, s_y, s_z) is scaling matrix in x, y, and z directions, **R** is rotation matrix, and **t** is translation vector

Affine Transformations

- Affine transformations preserve lines and parallel lines and are defined by expression

  x' = Ax + t

  where **A** is affine transformation matrix that can have any value, and **t** is translation vector

Affine Transformations

- For easier manipulation of matrix expressions, a representation using homogeneous coordinates is often used:
- Homogenous coordinate vector of a 3-D point is 4-D

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & t_1 \\ a_{21} & a_{22} & a_{23} & t_2 \\ a_{31} & a_{32} & a_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Projective Transformations

- Projective transformations are similar to affine (lines are preserved), only there is no preservation of parallel relation
- The analytical form is given by:

$$\mathbf{x}' = (\mathbf{Ax} + \mathbf{t}) / (\mathbf{p} \cdot \mathbf{x} + \alpha)$$

Projective Transformations

- In homogeneous coordinates we have:

$$\begin{bmatrix} u_1' \\ u_2' \\ u_3' \\ u_4' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & t_1 \\ a_{21} & a_{22} & a_{23} & t_2 \\ a_{31} & a_{32} & a_{33} & t_3 \\ p_1 & p_2 & p_3 & \alpha \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

where $x' = u_1' / u_4'$,
$y' = u_2' / u_4'$,
$z' = u_3' / u_4'$

Perspective Transformations

- Perspective transformations map 3-D space into a 2-D image plane
- Examples: camera imaging, X-ray imaging, microscopy
- Perspective transformations are a subset of projective transformations

Perspective Transformations

- For perspective transformations the affine part is often taken to be identity(**A** = **I**) and translation part to be zero (**t** = 0)
- Let **p** be the projection vector

$$\mathbf{p} = |\mathbf{p}|\hat{\mathbf{p}}$$

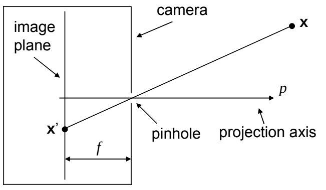
- From the general expression for perspective transformation we obtain:

$$\mathbf{x}' = f\mathbf{x} / (\hat{\mathbf{p}} \cdot \mathbf{x} + \alpha f)$$

where $f = 1/|\mathbf{p}|$

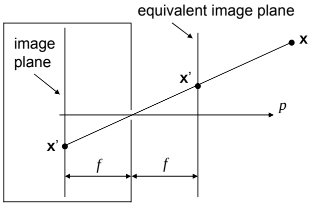
Perspective Transformations

- This expression is valid for a pinhole camera, where a small opening replaces a lens
- This is a good approximation of real world cameras



Perspective Transformations

- In the previous diagram, the image on the film is reversed, so sometimes an alternative representation is used where the image plane is located in front of the pinhole at the distance f



Perspective Transformations

- If the center of coordinate system is located at the pinhole, then $\alpha = 0$
- If the center of coordinate system is at intersection of the projection axis and image plane then $\alpha = 1$
- Parameter f is called focal distance

Elastic Transformations

- Elastic transformations do not preserve lines (i.e. a line can be mapped into a curve)
- An elastic transformation can be defined by any non-linear mapping of spatial coordinates
- Polynomial are often used in practice for simplicity
- For 3-D case:

$$\mathbf{x}' = \sum_{i=0}^{I} \sum_{j=0}^{J} \sum_{k=0}^{K} \mathbf{c}_{ijk} x^i y^j z^k$$

Elastic Transformations

- In practice, polynomial order is limited because of oscillations present in high-order polynomials
- For this reason, polynomial order is usually chosen so that $I, J, K \le 2$
- For the same reason it is often taken that $I + J + K \le 5$

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Image Registration Algorithms

- In the following material we present three groups of algorithms for 3-D medical image registration:
  - Algorithms using corresponding points identified in images
  - Algorithms using corresponding surfaces
  - Algorithms using voxel intensity values

Point-based Registration

- Point-based registration requires:
  - Identification of corresponding 3-D points in images to be aligned
  - Registration of the points (to identify unknown geometric transformation that maps one set of points into another)
  - Use determined transformation to map all other image points (i.e. to establish point correspondences)

Point-based Registration

- In medical image registration, 3-D points used for registration are often called fiducial markers or fiducial points
- 3-D points can be either external markers attached to human body or anatomical landmarks identified in the images

Point-based Registration

- The usual approach to point-based registration problem is to find the least-square rigid-body or affine transformation that aligns the points
- The obtained transformation can then be used to transform any point from one image to another
- This problem is often referred to as the orthogonal Procrustes problem

Procrustes

- Procrustes was a robber in Greek mythology, who offered visitors accommodation in his roadside house and a bed that would perfectly fit each visitor
- But Procrustes would put each visitor in the same bed and would either stretch the visitors, or cut off their bodies so they would fit the bed
- Luckily, the hero Theseus who traveled to Athens (and killed Minotaur on his way) stopped this practice by subjecting Procrustes to his own method

Orthogonal Procrustes Problem

- Problem definition: Given two configurations of N points in D dimensions $\mathbf{P} = \{ \mathbf{p}_i \}$ i $\mathbf{Q} = \{ \mathbf{q}_i \}$, it is necessary to find transformation T that minimizes error:

$$G(\mathrm{T}) = | \mathrm{T}(\mathbf{P}) - \mathbf{Q} |^2$$

- **P** i **Q** are $N \times D$ matrices whose rows are coordinates of points $\mathbf{p}_i$ i $\mathbf{q}_i$, and T(**P**) is the corresponding matrix of transformed points $\mathbf{p}_i$
- The standard case is when T is a rigid-body transformation

Orthogonal Procrustes Problem

- If T is affine transformation, we obtain the standard least-squares problem
- In the following material we show the solution for case when T is a rigid-body transformation defined by rotation matrix **R** and translation vector **t**

Solution

- First replace vectors in **P** and **Q** by their demeaned versions (mean value equal to zero):

$$\mathbf{p}_i \leftarrow \mathbf{p}_i - \bar{\mathbf{p}}, \quad \bar{\mathbf{p}} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{p}_i$$
$$\mathbf{q}_i \leftarrow \mathbf{q}_i - \bar{\mathbf{q}}, \quad \bar{\mathbf{q}} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{q}_i$$

- This reduces the problem to the orthogonal Procrustes problem in which we need to determine orthogonal rotation matrix **R**

Solution

- Central to the problem is the $D \times D$ correlation matrix $\mathbf{K} = \mathbf{P}^T\mathbf{Q}$, which shows how much the points in **Q** are predicted by points in **P**
- The singular value decomposition of matrix **K** is given by: $\mathbf{K} = \mathbf{U}\,\mathbf{D}\,\mathbf{V}^T$, where **U** and **V** are orthogonal matrices containing left and right singular vectors and **D** is a diagonal matrix containing singular values

Solution

- Orthogonal matrix **R** is determined by expression:

  $\mathbf{R} = \mathbf{V}\,\Delta\,\mathbf{U}^T$,

  where Δ = diag(1, 1, det ($\mathbf{VU}^T$))
- Translation vector **t** may be determined by expression:

  $\mathbf{t} = \overline{\mathbf{q}} - \mathbf{R}\overline{\mathbf{p}}$

Registration Errors

- Errors in rigid-body point registration are a result of:
  - Fiducial localization error (FLE), and
  - Fiducial registration error (FRE)
- The resulting error is called target registration error (TRE) is a result of both FLE and FRE
- If the FLE is ε the rotation and translation which solve the Procrustes problem will depend on ε :

  $T_\varepsilon = f\,(R_\varepsilon, t_\varepsilon)$
- TRE at the target **x** is then $|T_\varepsilon(\mathbf{x}) - T(\mathbf{x})|$ and it decreases with $1/N^{1/2}$

Surface Registration

- The second approach to 3-D registration is by using surfaces in medical images, which are often more distinct than point landmarks
- Segmentation algorithms are used to locate surfaces:
  - For example tissue to air boundaries often have high contrast, which makes surface detection easier
- If two correspondent surfaces can be detected in images to be matched, then rigid-body registration can be achieved by fitting the surfaces

Surface Registration Algorithms

- Some of the best known surface registration algorithms are:
  - The head and hat algorithm
  - Distance transform-based algorithms
  - Iterative closest point (ICP) algorithm

Head and Hat Algorithm

- Developed by Pelizzari i ostali, 1989, for 3-D registration of CT, MR and PET head images
- The first surface (head) is obtained from higher resolution modality and is represented as a stack of image slices
- The second surface (hat) is represented as a list of unconnected 3-D points
- Registration is performed by iterative transformation of the hat surface to find the best fit onto the head surface

Head and Hat Algorithm

- Registration accuracy is measured by the square of the distance between the point on the hat and the nearest point on the head in the direction of the head centroid
- Iterative optimization using Powell steepest descent algorithm, which performs a series of 1-D optimizations in each of the six dimensions:
  - For 3-D rigid-body registration we have six degrees of freedom (three rotations and three translations)
- This method is useful only for spherical surfaces

Distance Transforms

- Head and hat algorithm can be improved using a distance transform to preprocess head images
- A distance transform maps a binary image into a distance image
  - In the distance image each pixel has the value of the distance of that pixel to the nearest surface in the binary image
- Distance transform is computed for one of the images, which makes it easy to calculate distance from one surface to another

Iterative Closest Point Algorithm

- Iterative closest point (ICP) algorithm was developed by Besl and McKay, 1992, for 3-D registration
- It was developed for general use, but is now the most widely used surface matching algorithm for biomedical applications
- Let us assume that we have two surfaces:
  - The first surface (call it M) is the model surface
  - The second surface is given as a point set { $\mathbf{p}_i$ }

Iterative Closest Point Algorithm

- **repeat**
  - For each point $\mathbf{p}_i$ identify the closest point $\mathbf{q}_i$ on the model surface M
  - Use Procrustes method to register point sets $\mathbf{p}_i$ and $\mathbf{q}_i$
  - Apply identified geometric transformation to point set $\mathbf{p}_i$ to obtain the new set of points, call them $\mathbf{p}_i$'
  - Let $\mathbf{p}_i = \mathbf{p}_i$', for each *i*
- **until** the change in registration mean square error falls below a defined threshold

Intensity-based Methods

- In all previous methods registration was based on data extracted from images (landmark points or surfaces)
  - Advantage: It is not important how images are taken (different imaging modalities can be used) – registration is based on registration of points or surfaces
  - Disadvantage: A necessary segmentation step was required to extract landmark points or surfaces from images (which adds a possibility of error)

Intensity-based Methods

- Intensity-based methods use pixel or voxel values
  - Advantage: It is not necessary to have a separate segmentation step to extract points or surfaces of interest
  - A disadvantage is that images that are registered cannot be different (e.g. recorded using different imaging modalities)
- This approach is most natural when images of the same kind (same modality) are registered (e.g. CT to CT images)
- Intensity-based methods use various voxel similarity measures

Voxel Similarity Measures

- Voxel similarity measures are used to measure the accuracy of registration; the measure is based on pixel values
- Some of the most popular voxel similarity measures are:
  - Sum of the square intensity differences (SSD)
  - Correlation measures
  - Ratio image uniformity
  - Information theoretic measures

SSD Similarity Measure

- Let A and B be two images
- Sum of square intensity differences between images A and B is defined by:

$$\frac{1}{N}\sum_{x\in S}\left|A(x)-B(x)\right|^2$$

where N = card (S), and $S \subseteq Z^n$ is an overlap domain of image domains A and B

SSD Similarity Measure

- Disadvantage: SSD measure is very sensitive to a small number of pixels having large intensity differences in images A and B
- SSD measure assumes that the images to be registered differ only by Gaussian noise

Correlation Coefficient

- A less strict assumption is that there is a linear relationship between the intensity values in two images
- In this case the optimum measure is the correlation coefficient (CC)

Correlation Coefficient

- CC is defined by expression:

$$\frac{\sum_{x\in S}\left(A(x)-\bar{A}\right)\left(B(x)-\bar{B}\right)}{\left\{\sum_{x\in S}\left(A(x)-\bar{A}\right)^2\sum_{x\in S}\left(B(x)-\bar{B}\right)^2\right\}^{1/2}}$$

where $\bar{A}$ and $\bar{B}$ are the mean voxel values in images A and B, respectively, and S is an overlap of image domains A and B

Correlation Coefficient

- If A(x) = B(x) then CC = 1
- else CC < 1

Ratio Image Uniformity (RIU)

- Introduced by Woods for registration of serial PET images, but has also been used for serial MR registration
- The similarity measure uses a ratio image calculated from images A and B
- An iterative technique is used to find the geometric transformation that maximizes uniformity of the ratio image
  - Uniformity is measured by the normalized standard deviation of the voxels in the ratio image

Ratio Image Uniformity (RIU)

- When images A and B are registered the ratio image will be more uniform than when images are not registered
- The ratio image R(x) is calculated for voxels in the overlap S of images A and B

$$R(x) = \frac{A(x)}{B(x)}, \quad x \in S \qquad \overline{R} = \frac{1}{N}\sum_{x \in S} R(x)$$

$$\mathrm{RIU} = \frac{1}{\overline{R}}\sqrt{\frac{1}{N}\sum_{x \in S}\left(R(x) - \overline{R}\right)^2}$$

Information Theoretic Measures

- Image registration can be viewed as a problem of maximizing the amount of shared information in two images
  - When images are registered we just have two ears, two eyes, one nose, etc.
  - When images are out of alignment we have duplicate versions of these structures from images A and B (there is more information in images)
- This gives motivation to a different approach to registration, which is done by reducing the amount of information in two images

Information Theoretic Measures

- The idea: use a measure of information as a registration metric
- Information theoretic image similarity measures are based on the Shannon-Wiener entropy measure developed as a part of communication theory in 1940s
- This research area is called information theory

Entropy

- Let us assume that we have a source of information that sends messages coming from a set of N messages with probabilities p_i, $i = 1, \ldots, N$
- The amount of information in a single message with probability p_i may be measured by expression *log* p_i
- Entropy of an information source is by definition the average amount of information generated by the source:

$$H = \sum_{i=1}^{N} p_i \log p_i$$

Image Entropy

- Let us view an image as a source of information where each message is represented by a pixel (or voxel) having certain value
- The total number of messages that this information source generates is equal to the number of pixels
- Probability of each individual message (probability of a pixel having intensity *a* can be estimated using the first order image histogram *p*(*a*)

$$H(A) = \sum_{a} p(a) \log p(a)$$

Joint Entropy of Two Images

- Joint entropy of two images is a measure of the joint information contained in the images:

$$H(A,B) = \sum_{a}\sum_{b} p(a,b) \log p(a,b)$$

- If A and B are totally unrelated (i.e. statistically independent), then the joint entropy will be the sum of the entropies of the individual images (show proof for exercise):

$$H(A,B) = H(A) + H(B)$$

Joint Entropy of Two Images

- The following inequality holds for any two images:

$$H(A,B) \le H(A) + H(B)$$

- The more similar (i.e. less independent) the images are, the lower the joint entropy will be compared to the sum of individual entropies
- This is the motivation for use of joint entropy as an image similarity measure:
  - Registered images have low joint entropy
  - Unregistered images have larger joint entropy

Joint Entropy for Registration

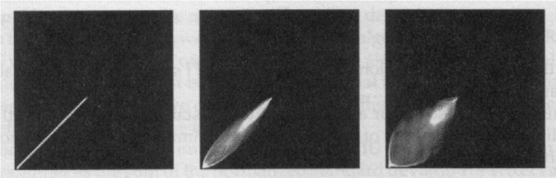
- To use joint entropy as an image similarity measure for registration, for each transformation *T*, we have to calculate joint entropy in the overlap domain of *T*(A) and B

$$H(A,B) = \sum_a \sum_b p^T(a,b) \log p^T(a,b)$$

- The superscript *T* in the above expression emphasizes that the PDFs $p(a,b)$ change with *T*

Example of 2nd Order Histograms

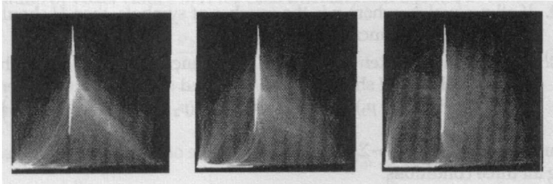
- For two identical MR images of the head:
  - Left: Accurate registration
  - Middle: 2mm translation of one image w.r.t. another
  - Right: 5 mm translation of one image w.r.t. another



From Hajnal et al., Medical Image Registration, CRC Press, 2001

Example of 2nd Order Histograms

- For CT and MR head images:
  - Left: Accurate registration
  - Middle: 2mm translation of one image w.r.t. another
  - Right: 5 mm translation of one image w.r.t. another



From Hajnal et al., Medical Image Registration, CRC Press, 2001

Joint Entropy Limitations

- A registration algorithm that minimizes the joint entropy may in certain cases converge to a wrong solution
- The second limitation is that interpolation algorithms (which typically blur images) will change joint PDF and therefore will change joint entropy

Mutual Information

- A solution to the overlap problem of joint entropy is to measure the information contributed to the overlapping volume by each image registered and by joint entropy
- This can be done using a measure of mutual information (MI) introduced by Shannon in 1948

Mutual Information

- Mutual information between images A and B (or any two sources of information) is defined by:

$$I(A,B) = H(A) + H(B) - H(A,B)$$

- Mutual information is a similarity measure that shows how well one image describes another (in the sense how well can we estimate one image based on another)

Mutual Information

- For two statistically independent images it holds that

$H(A,B) = H(A) + H(B)$

therefore

$I(A,B) = H(A) + H(B) - H(A,B) = 0$

Normalized Mutual Information

- In certain cases MI does not behave well
- For example, changes in overlap of low intensity region can disproportionately change MI
- For this reason alternative normalizations of joint entropy have been proposed to overcome the problem of MI sensitivity to change in image overlap

Normalized Mutual Information

- Maes at al. have proposed the following measures:

$$I_1(A,B) = \frac{2I(A,B)}{H(A)+H(B)}$$

$$I_2(A,B) = H(A,B) - I(A,B)$$

Normalized Mutual Information

- Studholme has proposed the following image similarity measure:

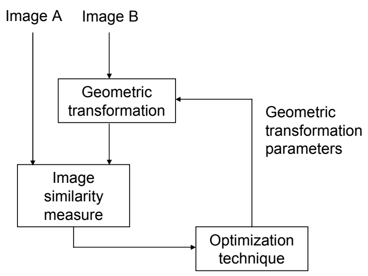
$$I_3(A,B) = \frac{H(A)+H(B)}{H(A,B)}$$

- This measure has shown to be much more robust than standard MI

Numerical Optimization

- For most registration techniques, unknown parameters of geometric transformation are determined using an iterative numerical optimization method
- The goal function which is optimized (maximized) is an image similarity measure

Image Registration



Numerical Optimization

- The optimization of an image similarity measure is done with respect to free parameters of geometric transformation:
  - For 3-D rigid body transformation we have 6 parameters
  - An affine transformation has 12 free parameters
  - Elastic transformations may have many more parameters (hundreds or thousands)
- Large (multidimensional) optimization spaces make optimization problem more difficult

Numerical Optimization: Problems

- The goal of optimization is to find the global minimum
- With a large optimization space there is a problem of getting stuck in a local minimum
- Large optimization sizes result in high complexity and long execution time (problem for practical clinical applications)
- Multiresolution approaches may be used to fight this problem

Conclusion

- Image registration has many applications in biomedical imaging and in other areas
- Intensive research during the last decade
- We have covered some basic aspects and problems of image registration

- For further information consult the literature

Literature

- M. Sonka, J. M. Fitzpatrick, Eds., Handbook of Medical Imaging, Volume 2. Medical Image Processing and Analysis, SPIE, 2000
- J. V. Hajnal, D. L. G. Hill, D. J. Hawkes, Eds., Medical Image Registration, CRC Press, 2001

Thank you for your attention